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Dose response Hill model for the arsenic contamination in the groundwater samples of the Lakhimpur district of Uttar Pradesh

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Abstract: An earlier experimental study by Ankit (see, Ankit Kumar, Vandana Gupta, Jai Gopal Pandey, Shikha Govil, and Rakesh Patel, "Status of arsenic contamination at district Lakhimpur (Uttar Pradesh), India" in Emerging Trends in Science, Social Science and Engineering, Sudhanshu Aggarwal, Rameshwar Pandey, Puja Naik, Ajay Kumar Mishra, Khushboo Raj, Tripuresh Kumar Tripathi and Sudhir Kumar Shukla (Editors), 60-73, ISBN 9789358380125, Astitva Prakashan, Nehru Nagar, Bilaspur, Chhattisgarh -495001, India) has found that the arsenic contamination of the groundwater resources at eight selected study sites of the Lakhimpur district of Uttar Pradesh is higher at lesser depths from the surface level, i.e., in the shallow region as compared to its corresponding values at greater depths from the surface, i.e., in the deep India Mark II region. We propose to fit a dose response Hill model in this paper to account for the pattern of arsenic contamination of the groundwater resources of Lakhimpur district based on this report of Ankit Kumar et. al. (op. cit.).

Keywords: arsenic; arsenic contamination; regression; DR Hill model; Hill slope; regression

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1. Introduction

Potable water for use by the living organisms in this planet has become an extreme necessity today, when there is a hue and cry everywhere to save the mother earth from pollution and therefore the life, which is threatened by uncontrolled human industrial activity in the name of development that has very seriously jeopardized the very existence of life on this planet. Water, a mandatory prerequisite of life, does not remain unaffected with this horrible phenomenon of pollution across the globe. The whole world today is trying to save the sources of water and air from pollution so that the various forms of life existing on the earth are sustained for the future generations and this planet remains habitable for all. There is no need for us to stress here that a number of studies are currently being undertaken by various researcher across different disciplines to study the harmful impact of the rising levels of pollutions of the air and water resources of the earth and to arrive at meaningful conclusions so that the further degradation of the quality of the air and the water resources of the earth, both of which are extremely important to maintain the existence of life, may be immediately stopped. Arsenic is one of the main contaminants which are increasingly polluting the water resources of this planet. While addressing the United Nations on the World Water Day – on 22nd March, 2001 the erstwhile Secretary General of the United Nations, (see, [1], p. 6 of 173):

Access to safe water is a fundamental human need and, therefore, a basic human right. Contaminated water jeopardizes both the physical and social health of all people. It is an affront to human dignity. Yet even today, clean water is a luxury that remains out of the reach of many."

These most deeply concerning words are extremely relevant even today after more than two decades, when the problem of contamination of the natural resources of air and water are assuming alarming proportions with each passing day and the concerted global efforts aimed at their remedial are proving highly insufficient to grapple this monster. Millions of people around the world today are forced by circumstances to consume water that has toxic concentrations of arsenic far above the maximum permissible level of 10 microgram per liter as the standard set up by the World Health Organization (see, [1], p. 5 of 173). Based on studies it is established that the primary cause of contamination of groundwater by arsenic is primarily due to natural (geological) reasons but it is also due to human industrial activities like, metal mining and smelting ([1], p. 5 of 173). Chronic intake of arsenic contaminated water by humans is reported to result in skin lesions, diseases of the peripheral nervous system, damage to liver, circulatory diseases, anemia, cancer, etc. (see, [1], p. 4, i.e., p. 12 of 173). Arsenic contamination of groundwater resources has become a global phenomenon. The work [2] attributes this to a variety of geochemical process which include the oxidation of minerals containing arsenic sulfide, desorption of arsenic from surfaces containing oxide and hydroxides of this metal, arsenic releases from geothermal water sources, etc (see also, [1], p. 32 of 173). Authors [3] found another cause for this – percolation (leaching) of arsenic from sulfides by carbonate ions into the aquifers (see also, [1], p. 32 of 173).

Concentrating on the Indian scenario about arsenicosis, we cite here the following passage from the Guidelines on Arsenicosis ([4], p. 4 of 26) issued by the Ministry of Health and Family Welfare, Government of India, which aptly highlights some of the causes of arsenicosis in our country:

Shift from surface water and shallow open well sources to deep tube well in Arsenic affected areas have also led to Arsenic contamination in some States. Arsenic leaching may also occur from industrial sources or from Arsenic containing insecticides, herbicides or rodenticides.

People are exposed to elevated levels of inorganic Arsenic by drinking and using contaminated water (mostly groundwater) for cooking, irrigation of food crops, industrial processes and smoking tobacco.

Arsenic is also found in groundwater at an average of between 10m and 60m depth and deeper groundwater is generally free from Arsenic."

The studies [5–8] have found that the regions of our country in West Bengal and the adjoining Bangaldesh, where public water supply is drawn from aquifers situated in the Gangetic basin, naturally occurring arsenic is the main pollutant of these aquifers, which is released into these sources due to the oxidation of the natural deposits of arsenic pyrites in this region by the atmospheric oxygen which can directly interact with these aquifer sediments as a direct consequence of the lowering of the water level by abstraction (see also, [1], p. 32 of 173). The presence of arsenic in the groundwater reservoirs of northern India and its connection to liver diseases was first of all brought to light in the works [9] and [10]. The arsenic contamination of the groundwater sources around the basin of the river Ganga in Bihar was pointed out by [11] (see also, [12] , p. 352 of 617 and p. 353 of 617). The study conducted about the level of arsenic contamination in the Nadia district of West Bengal by [13] found that out of the surveyed population 15.43% revealed the symptoms of arsenicosis. They also noted that the problem of arsenic contamination of water resources also has a socio-economic dimension because the majority of the people manifesting arsenicosis came from the lower socio-economic strata of the society living in abject conditions who were engaged in physical labor or farming and not adequately educated. Chronic lung diseases, arsenical skin lesions and peripheral neuropathy were also detected among the subjects of the survey. The lack of awareness and inadequate health care support facilities added further to the misery of the severely arsenic affected people. The authors of ([14], p. 2 of 63) mention that in India twenty states and four union territories are facing the problem of arsenic contamination of groundwater. They also remark that 90% of the arsenic contamination of water resources is due to geogenic phenomena, and identify the alluvial sediments as the chief sources of arsenic contamination. Four major geological processes which are said to be at the root of natural arsenic poisoning of groundwater are postulated by ([14], p. 2 of 63) to be plate tectonic processes, mountain building, erosion and sedimentation. Since arsenic contamination of water poses a serious health hazard for a majority of human population across the world and in India too. The study of arsenic contamination assumes much importance against this backdrop.

Drawing our inspiration from the above studies, we propose to begin our studies of this topic from the mathematical and statistical point of view. For this paper we gratefully take the data reported in the study [15] as our starting point, which brings forth the experimentally measured levels of arsenic contamination in the groundwater resources of the Lakhimpur district of Uttar Pradesh province of India. After reproducing the data of [15] in Table 1 in §2 of the paper, we resort to regression technique to study the behavior of arsenic contamination as reported in this study [15] and propose a Dose Response Hill (DR Hill) Model to explore the possible pattern of contamination. In §1.1 below we give a brief description of the DR Hill model for the sake of completeness and quick reference.

In §3 we present the mathematical analysis of our model and the conclusion of the study is summed up in §4.

1.1. The dose response Hill model

The celebrated Hill equation that we propose to fit to the data of [15] in this paper was originally given by [16,17] and is a very widely used equation (model) to measure the response of a target (e.g., a cell, muscle, tissue or an organism) as a function of the exposure of the target to a stimulus (like a chemical, drug, or an external agent, like radiation to a photoreceptor, sound to an otoreceptor, etc.) (see, [18]) in biochemistry and pharmacology. The Hill equation belongs to the general class of models called the Sigmoidal curves (characterized by their S shaped curve), which are practically flat at their beginnings and ends and show the steepest descent in their middle region. It is on the basis of these models researchers in pharmaceutics and developers of new drugs, chemicals, etc. determine the safe, hazardous (lethal) doses of drugs, pollutants, poisons, toxicants, fertilizers, etc. to which humans, other animals and plants are exposed [18]. In the dose –response curves, the administered stimulus (dose) or its logarithm is plotted on the x - axis and its response is plotted on the y - axis. Typically, a Hill equation is written in the form:

$$E = \frac{E[A]^n{}_{max}}{EC_{50}^n + [A]^n}, \tag{1}$$

where, E is the magnitude of the response, E_{max} is the maximal response, EC_{50}^n denotes the stimulus value (e.g., drug concentration) which produces 50% of the maximal response in the subject, which also represents the *point of inflection of the dose response curve* (e.g., see [19], p. 68) and n is called the *Hill coefficient* (see, [18,20]) or the *Hill slope* ([19], p. 68), which represents the slope at the steepest part of the curve ([19], p. 68). The first point on the plot of a Hill equation, where a response above zero is obtained is called the *threshold value* of the dose, and it is for values slightly above this value of the concentration of a drug (stimulus), its beneficial effects (good response) are visible (i.e., during clinical trials the efficacy of a drug for the treatment of an ailment in a subject (patient) is decided by this value and the appropriate doses to be administered for a successful treatment of an ailment are based on this criterion besides other factors). Administering concentrations of a stimulus (drug) too high above those predicted by the steepest range of its Hill plot do not result in any further improvement in the value of the response produced (because as noted earlier, the sigmoidal curves are flatter both at the initial and final stages of their range) and it may also produce unwanted or unfavorable response (like the undesirable side effects of the drug in some subjects (patients)) (see, [18,20]). It may be noted that in (1) there are *three parameters* - EC_{50max}^n and n in the Hill equation. However, we shall fit a *four parameter* Hill equation given by (2) below to our dataset in this paper. A notable mulitphasic model of the Hill equation is also recently proposed in [21].

2. Data for the study

In Table 1 below we present the secondary data which we have used for this study. This data was published in the work [15]. We thankfully and sincerely acknowledge that source and all the authors of [15].

Table 1. Arsenic contamination in parts per billion in the underground water sources of the various sites of the Lakhimpur district of Uttar Pradesh, [15]

S. No.	Name of the study site	Deep (India Mark II) ppb. (parts per billion)	Shallowppb. (parts per billion)
1.	Durga Purwa	48	63
2.	Dudhwa Range Colony	47	58
3.	Govind Dhaniram Purwa	57	61
4.	Majar Majhgai	53	62
5.	Naugara	56	62
6.	Sajai Purwa	43	51
7.	Fateh Singh Academy	46	53
8.	Sisaiya	49	58

3. DR Hill regression model for the arsenic contamination of the samples of groundwater sources of the Lakhimpur district

Our studies of the secondary data of the experiments of [15] led us to conclude that among a host of other possible models, the Dose Response Hill Model can be a suitable model to mathematically explain the results of this study [15]. In this section below we give the details of our proposed DR Hill Model. Since it is evident from Table 1 that for the eight samples of groundwater collected from different places in the Lakhimpur district, the arsenic contamination levels in parts per billion are found higher in the Shallow region as compared to those in the Deep (India Mark II) (for short, Deep) region, we decided to take the arsenic concentration in the Deep region as our predictor (independent) variable and that in the Shallow region as our response (dependent) variable. We mention here the elementary fact that for the sample of size eight given in Table 1, a seventh degree polynomial would provide an exact fit, if we talk about polynomial regressions in the general family of linear regression. Instead of that we decided to search the nonlinear regression category and found that among the family of Dose-Response Models the DR Hill Model is a good model which can explain with a reasonably good accuracy the phenomenon of arsenic contamination as uncovered by the results of the work of [15]. For the sake of the interested readers we remark that among the vast variety of literature available on the usefulness of regression analysis as an important research tool we are just mentioning only a handful of references [22–30] on this topic here, the details of which may be enquired into by the researchers themselves.

For the accuracy of our computations we scale the data of Table 1 by a factor of 0.01 and then put forward our proposed DR Hill model in (2) below which accounts well for the observed trend of arsenic contamination displayed in Table 1:

$$y = \alpha + \frac{\theta x^{\eta}}{\kappa^{\eta} + x^{\eta}}, \tag{2}$$

where, in (2) y denotes the response (Shallow, i.e., the arsenic contamination in $\times 10^2$ ppb in the shallow region) and x represents the predictor (Deep, i.e., the arsenic contamination in $\times 10^2$ ppb in the deep region) and $\alpha, \theta, \kappa, \eta$ are the *four* regression parameters. In the terminology of §1.1 κ denotes the EC_{50}^{η} and η the Hill slope or the Hill coefficient of the model (2).

The necessary details of the model of (2) are displayed in Table 2 below and its covariance matrix is displayed in Table 3. The degrees of freedom (DOF) are 4 and the Akaike Information Criterion (corrected) AICc has a value -56.130235. With a high value of the coefficient of determination at 0.884, the model seems reliable. Figure 1 shows the plot of the DR Hill model of (2) with the data points and the 95% confidence band (narrower darker red colored) and the prediction band (wider light red colored). It is at once evident from this figure that seven out of the eight sample observations of Table 1 lie well within the 95% confidence band and only one data point lies in the 95% prediction band that too near the lower boundary of the confidence band, which represents the point (49 ppb, 58 ppb) corresponding to Sisaiya in Table 1. The four points which lie almost exactly on the plot of (2) (i.e., the red colored line in Figure 1) are the points (43 ppb, 51 ppb) Sajai Purwa, (46 ppb, 53 ppb) Fateh Singh Academy, (47 ppb, 58 ppb) Dudhwa Range Colony and (57 ppb, 61 ppb) Govind Dhaniram Purwa.

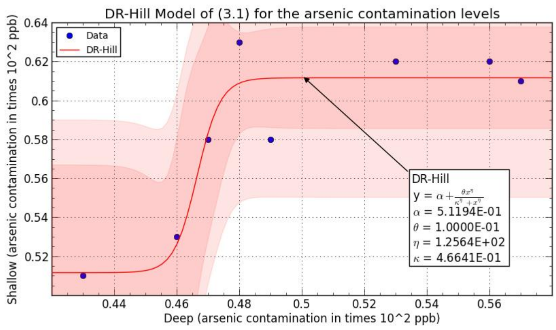

Figure 1. The Plot of the DR-Hill Model of (2) for the sample of Table 1 in which the arsenic concentration is in $\times 10^2$ ppb

The residuals of the proposed DR Hill Model are shown in Figure 2 below. They show a random pattern of distribution around the Zero line indicating a good fit. The number of runs reported is 7 and the P Value of the Wald-Wolfowitz runs test conducted on the residuals being 0.8740 (which being well above the threshold of 0.05) indicates that the pattern is not unlikely.

Table 2. Details of the DR Hill Model of (2)

Name: DR-Hill
Kind: Regression
Family: Dose-Response Models
Equation: $\alpha + \theta * x \wedge \eta (\kappa \wedge \eta + x \wedge \eta)$
No. of Independent variables: 1
Parameters:
α = 5.11935400206815E-01
θ = 1.00004168868008E-01
η = 1.25644857566274E+02
κ = 4.66409845499895E-01
Value Std Err Range (95% confidence) α 0.511935 0.019910 0.456655 to 0.567215 θ 0.100004 0.022450 0.037674 to 0.162334 η 125.644858 97.862061 -146.063782 to 397.353498 κ 0.466410 0.004097 0.455034 to 0.477785
Standard Error: 2.00091400141943E-02
Coefficient of Determination (r∧2): 8.83951975620976E-01
Correlation Coefficient (r): 9.40187202434162E-01
DOF: 4
AICc: -56.130235
Parameter Standard Deviations:
α_stddev = 1.99103755072080E-02
θ_stddev = 2.24495124837753E-02
η_stddev = 9.78620609120732E+01
κ_stddev = 4.09719962916691E-03
Parameter Uncertainties, 95%:
α_unc = 5.52800646196377E-02
θ_unc = 6.23298390496548E-02
η_unc = 2.71708640003895E+02
κ_unc = 1.13756498554187E-02

Table 3. Covariance matrix of the DR Hill Model of (2)

	α	θ	η	κ
α	9.9015242458056241E-01	-1.0142716202806958E+00	2.3538336134271676E+03	1.1885332496101297E-01
θ	-1.0142716202806961E+00	1.2588007183545140E+00	-2.9107894863618899E+03	-9.7672314580624947E-02
η	2.3538336134271685E+03	-2.9107894863618903E+03	2.3920588966819931E+07	3.4230429787212483E+02
κ	1.1885332496101289E-01	-9.7672314580624892E-02	3.4230429787212461E+02	4.1929279824922672E-02

Figure 3 depicts the Convergence History (or the Residual History) Plot of the model of (2) in which the two quantities, viz., the norm of the residual (which is the difference between the fitted values and the actual data values) and the change in the residual are plotted as a function of the number of iterations performed during our experimentation process. In our experiments we preset the tolerance limit at 10^{-8} for both the change in residual and the change in parameters and the maximum number of iterations set by us was 100 after which the process of computation would get terminated if the desired convergence was not achieved till that stage. The residuals (red colored line in Figure 3) have attained an almost steady state about after the fifth iteration and this state continue almost till the thirtieth iteration and after the thirty fifth iteration there is no change in the residual. The residual change (the blue colored line in Figure 3) has fallen below the preset tolerance of 10^{-8} after approximately the thirty eighth iteration indicating the convergence of our iteration process.

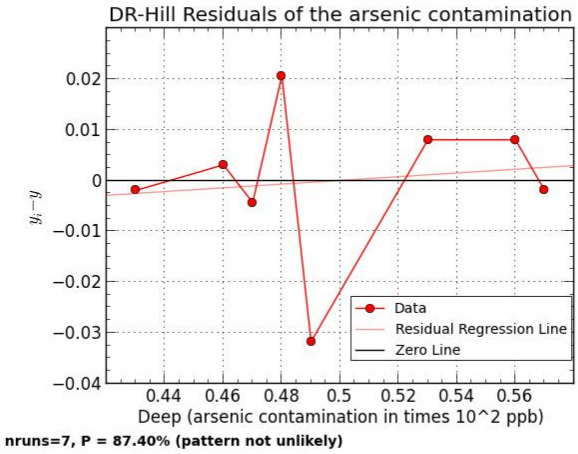

Figure 2. The Residual Plot of the DR Hill Model of (2)

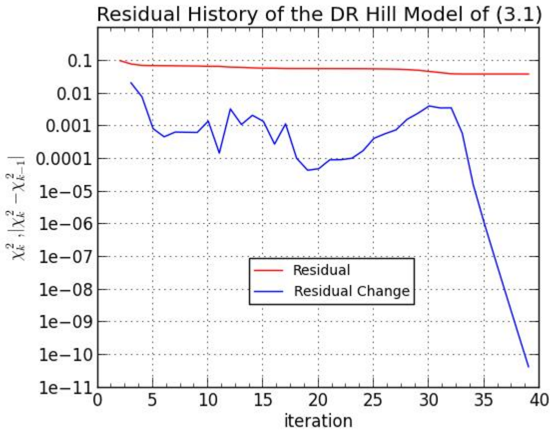

Figure 3. The Residual History Plot of the DR Hill Model of (2)

The Parameter Histories Plot of Figure 4 for our model at once shows that the values of the parameters α (the red colored line) κ (the pink colored line) remain the same (i.e., constant) right from the very beginning of our experiment. The value of the parameter θ (the green colored line) shows a totally insignificant variation between the zeroth and the first iteration, (which we ought to have ignored, but we mention it here only for the purpose of drawing the attention of the inquisitive reader towards even the minutest detail of our curve fitting experiment) after which as we can observe that this parameter value has also become invariant. The most striking feature of Figure 4 is the blue colored line showing the variation in the values of the parameter η (the Hill coefficient or the Hill slope) as the number of iterations increase with the progress of the experiment. The large fluctuations in the values of this parameter η with minor variations during the first about three iterations and then showing a slow increase in the values of the parameter with the ongoing corresponding increase in the number of iterations and then a very steep rise in the parameter value of η from about 15 at the twenty sixth iteration to a value beyond 120 and near 125 at about the thirty second iteration and after that its value settles rapidly to its ultimate value of 125.645 after the thirty fifth iteration which is *of course the most intriguing and astonishing feature of this computation process of ours for which we do not find ourselves to be in a position to explain this unusual behavior of the Hill coefficient parameter η!*. One plausible cause to which we ascribe this behavior, which intuitionally becomes evident to us, is that a look at (2) immediately shows us that the parameter η is the power of the predictor x as well as the parameter κ both in the numerator and denominator of the second fraction occurring on the right side of (2), which, in our view, definitely speaks of the predominantly important role of this parameter vis-à-vis the other three parameters of this model! The precise conclusion of the analysis of Figure 4 is that the all the four parameters of the model described by (2) have attained constant values, which is one of the most desired features of a fitting process for it to produce reliable predictions and a reasonable fit to the data being studied. The parameter κ here obviously corresponds to the EC_{50}^{n} value already mentioned in the §1.1.

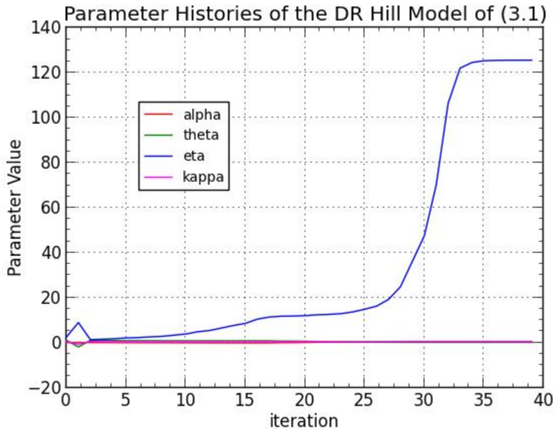

Figure 4. The Parameter Histories Plot of the DR Hill Model of (2)

The values of the response (y Shallow) predicted by the DR Hill Model of (2) for representative values of the predictor (x Deep) at equally spaced intervals of 0.0016 $\times 10^2$ ppb are shown by us in Table 4, where the value of 4.1999999999999998E-01 means 4.1999999999999998 $\times 10^{-1}$, etc.

Table 4. A representative prediction table for the DR-Hill Model of (2) for the arsenic contamination levels in the Shallow region as a function of that in the Deep (India Mark II) region

S. No.	x = Deep (India Mark II) (Arsenic contamination in $\times 10^2$ ppb)	y = Shallow (Arsenic contamination in $\times 10^2$ ppb)
1.	4.1999999999999998E-01	5.1193559113146359E-01
2.	4.2159999999999997E-01	5.1193570805764532E-01
3.	4.2319999999999997E-01	5.1193589569402076E-01
4.	4.2479999999999996E-01	5.1193619626368492E-01
5.	4.2639999999999995E-01	5.1193667688327804E-01
6.	4.2799999999999994E-01	5.1193744405226416E-01
7.	4.2959999999999993E-01	5.1193866646640185E-01
8.	4.3119999999999992E-01	5.1194061087356046E-01
9.	4.3279999999999991E-01	5.1194369832531805E-01
10.	4.3439999999999990E-01	5.1194859225907896E-01
11.	4.3599999999999989E-01	5.1195633614561242E-01
12.	4.3759999999999988E-01	5.1196856812145974E-01
13.	4.3919999999999987E-01	5.1198785482729381E-01
14.	4.4079999999999986E-01	5.1201820911008333E-01
15.	4.4239999999999985E-01	5.1206588983887191E-01
16.	4.4399999999999984E-01	5.1214063141137489E-01
17.	4.4559999999999983E-01	5.1225752068880215E-01
18.	4.4719999999999982E-01	5.1243983344547350E-01
19.	4.4879999999999981E-01	5.1272325600066893E-01
20.	4.5039999999999980E-01	5.1316202076667761E-01
21.	4.5199999999999979E-01	5.1383748540003971E-01
22.	4.5359999999999978E-01	5.1486935691641578E-01
23.	4.5519999999999977E-01	5.1642862275472678E-01
24.	4.5679999999999976E-01	5.1874845932351465E-01
25.	4.5839999999999975E-01	5.2212396284274398E-01
26.	4.5999999999999974E-01	5.2688376600655917E-01
27.	4.6159999999999973E-01	5.3331178610024843E-01

Continued on next page

Table 4 – continued from previous page

S. No.	Deep (India Mark II) (Arsenic contamination in ppb)	Shallow (Arsenic contamination in ppb)
28.	4.6319999999999972E-01	5.4151034947474785E-01
29	4.6479999999999971E-01	5.5124412893218289E-01
30.	4.6639999999999970E-01	5.6187117491632388E-01
31.	4.6799999999999969E-01	5.7246889193199224E-01
32.	4.6959999999999968E-01	5.8213040328923671E-01
33.	4.7119999999999967E-01	5.9024868342388515E-01
34.	4.7279999999999966E-01	5.9661918954659920E-01
35.	4.7439999999999966E-01	6.0135759130944766E-01
36.	4.7599999999999965E-01	6.0474526677105200E-01
37.	4.7759999999999964E-01	6.0710050050581732E-01
38.	4.7919999999999963E-01	6.0870710000178041E-01
39.	4.8079999999999962E-01	6.0978938909804759E-01
40.	4.8239999999999961E-01	6.1051268824909299E-01
41.	4.8399999999999960E-01	6.1099373123889422E-01
42..	4.8559999999999959E-01	6.1131277575124399E-01
43	4.8719999999999958E-01	6.1152408879749076E-01
44..	4.8879999999999957E-01	6.1166398625745799E-01
45.	4.9039999999999956E-01	6.1175661921316915E-01
46.	4.9199999999999955E-01	6.1181799062150632E-01
47.	4.9359999999999954E-01	6.1185868403520183E-01
48.	4.9519999999999953E-01	6.1188569328928333E-01
49	4.9679999999999952E-01	6.1190363973154827E-01
50.	4.9839999999999951E-01	6.1191557826978049E-01
51.	4.9999999999999950E-01	6.1192352977515618E-01
52.	5.0159999999999949E-01	6.1192883231492068E-01
53.	5.0319999999999943E-01	6.1193237278516277E-01
54.	5.0479999999999947E-01	6.1193473970330259E-01
55.	5.0639999999999952E-01	6.1193632405472986E-01
56.	5.0799999999999945E-01	6.1193738590948321E-01
57..	5.0959999999999939E-01	6.1193809847111913E-01
58.	5.1119999999999943E-01	6.1193857723490952E-01
59.	5.1279999999999948E-01	6.1193889931147483E-01
60.	5.1439999999999941E-01	6.1193911624821440E-01
61.	5.1599999999999935E-01	6.1193926254684949E-01
62.	5.1759999999999939E-01	6.1193936132879179E-01
63.	5.1919999999999944E-01	6.1193942810809077E-01
64	5.2079999999999937E-01	6.1193947330721488E-01
65.	5.2239999999999931E-01	6.1193950393664420E-01
66.	5.2399999999999936E-01	6.1193952471759305E-01
67.	5.2559999999999940E-01	6.1193953883341901E-01
68.	5.2719999999999934E-01	6.1193954843313902E-01
69..	5.2879999999999927E-01	6.1193955496924501E-01
70.	5.3039999999999932E-01	6.1193955942462552E-01
71.	5.3199999999999936E-01	6.1193956246517900E-01
72.	5.3359999999999930E-01	6.1193956454257670E-01
73.	5.3519999999999923E-01	6.1193956596353971E-01
74..	5.3679999999999928E-01	6.1193956693659846E-01
75.	5.3839999999999932E-01	6.1193956760369028E-01
76.	5.3999999999999926E-01	6.1193956806153627E-01
77.	5.4159999999999919E-01	6.1193956837612118E-01
78.	5.4319999999999924E-01	6.1193956859251131E-01
79.	5.4479999999999928E-01	6.1193956874152167E-01
80..	5.4639999999999922E-01	6.1193956884424527E-01
81	5.4799999999999915E-01	6.1193956891513757E-01
82.	5.4959999999999920E-01	6.1193956896411505E-01
83.	5.5119999999999925E-01	6.1193956899798863E-01
84..	5.5279999999999918E-01	6.1193956902144131E-01
85.	5.5439999999999912E-01	6.1193956903769631E-01
86.	5.5599999999999916E-01	6.1193956904897440E-01
87.	5.5759999999999921E-01	6.1193956905680780E-01
88.	5.5919999999999914E-01	6.1193956906225422E-01

Continued on next page

Table 4 – continued from previous page

S. No.	Deep (India Mark II) (Arsenic contamination in ppb)	Shallow (Arsenic contamination in ppb)
89..	5.6079999999999908E-01	6.1193956906604496E-01
90.	5.6239999999999912E-01	6.1193956906868607E-01
91	5.6399999999999917E-01	6.1193956907052816E-01
92..	5.6559999999999910E-01	6.1193956907181413E-01
93.	5.6719999999999904E-01	6.1193956907271296E-01
94	5.6879999999999908E-01	6.1193956907334168E-01
95.	5.7039999999999913E-01	6.1193956907378200E-01
96.	5.7199999999999906E-01	6.1193956907409064E-01
97.	5.7359999999999900E-01	6.1193956907430724E-01
98.	5.7519999999999905E-01	6.1193956907445934E-01
99.	5.7679999999999909E-01	6.1193956907456626E-01
100.	5.7839999999999903E-01	6.1193956907464153E-01
101.	5.7999999999999896E-01	6.1193956907469460E-01

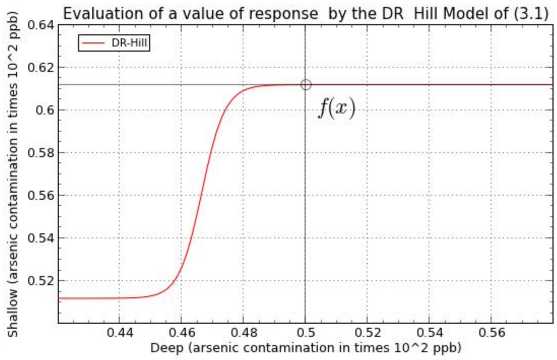

Figure 5. Evaluation of a value of the response for a given value of the predictor by the DR Hill Model of (2)

A representative evaluation of the response $y = Shallow$ for a given value of the predictor $x = Deep$ is presented in Figure 5 above, where corresponding to $x = 0.5 \times 10^2$ ppb, we get $y = 0.611923529775 \times 10^2$ ppb. We also mention that for a given value of the response $y = Shallow$we can also recover the corresponding value of the predictor $x = Deep$ from (2). As an illustration we present in Table 4 below a set of selected given values of the response $y = Shallow$corresponding to which we evaluate the values of the predictor $x = Deep$ using (2) with an initializing value of the predictor $x0 = 0.46 \times 10^2$ ppb.

Table 5. Recovering the values of the predictor $x = Deep$ from the given values of the response $y =$ Shallow for the DR Hill Model of (2) with $x0 = 0.46 \times 10^2$ ppb

S. No.	$y =$ Shallow (Arsenic contamination in $\times 10^2$ ppb)	$x =$ Deep (India Mark II) (Arsenic contamination in $\times 10^2$ ppb) with $x\,0 = 0.46 \times 10^2$ ppb
1.	5.1193559113146359E-01	0.419999999999
2.	5.1195633614561242E-01	0.436
3.	5.1383748540003993E-01	0.452
4.	5.7246889193199435E-01	0.468
5.	6.1099373123889433E-01	0.484
6.	6.1192352977515618E-01	0.5
7.	6.1193926254684949E-01	0.515999999999
8.	6.1193956246517900E-01	0.531999999991
9.	6.1193956891513757E-01	0.548000002026
10.	6.1193956907052816E-01	0.564000057845
11.	6.1193956907469460E-01	0.5800011295

Figure 6 gives an evaluation of the slope of the curve of the DR Hill Model of (2), where we find that at $x = 0.46 \times 10^2$ ppb we have $f'(x)\,|_{x=0.46} = \frac{dy}{dx}|_{x=0.46} = 3.47269345224$. The total area under the curve of (2) is evaluated as $\int_{0.43}^{0.57} f(x)\,dx = 0.0820255304145(\times 10^2\text{ppb})^2$ which is displayed in Figure 7. The total arc length under the curve of (2) is calculated to be $\int_{0.43}^{0.57} \sqrt{1 + (f'(x))^2} dx = 0.214211553115 \times 10^2$ppb. This arc length is shown in the Figure 8.

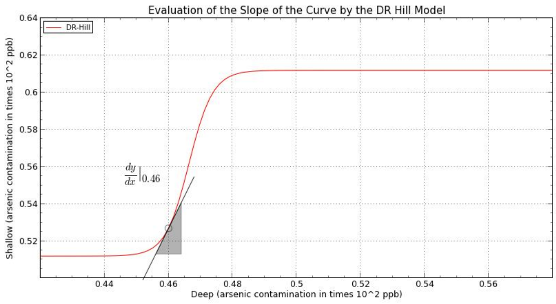

Figure 6. Evaluation of the slope of the curve for a given value of the predictor by the DR Hill Model of (2)

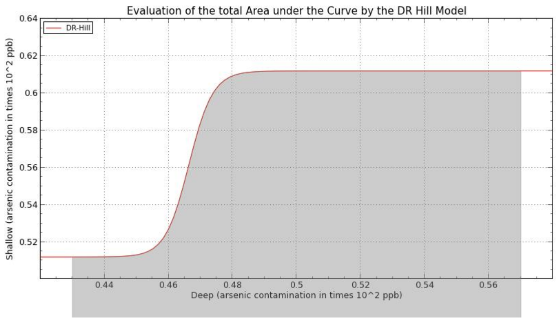

Figure 7. Evaluation of the total area under the curve for a given value of the predictor by the DR Hill Model of (2)

In Table 6 we present a representative analysis table of the model of (2) for given values of the predictor $x =$ Deep ranging from $x = 4.1999999999999998E - 01 \times 10^2$ppb to $x = 5.8000000000000007E - 01 \times 10^2$ppb at equal distances of $x = 0.016 \times 10^2$ppb.

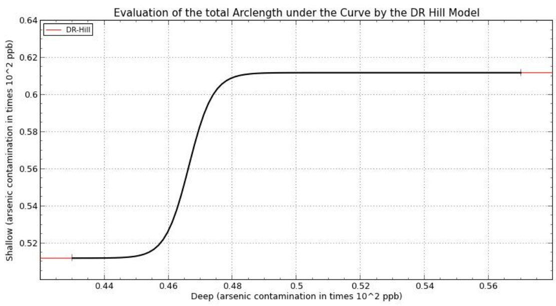

Figure 8. Evaluation of the total arc length under the curve for a given value of the predictor by the DR Hill Model of (2)

4. Conclusion

We applied the Dose Response Hill model of (2) to study the pattern of arsenic contamination in the groundwater sources of the Lakhimpur district of Uttar Pradesh. The model is one of many other possible models which can be employed successfully to discern the pattern of arsenic contamination in the area under study. The various prediction tables based on this model are also given by us including the corresponding figures of the model fitted by us to the dataset of Table 1. When the safety of the groundwater resources of the country and the world as a whole is concerned and prevention of their contamination from poisonous substances, industrial effluents, etc. has become a prime concern for the survival of life on the earth, these types of studies also assume much significance. In our forthcoming papers we shall also present our many other analyses on the sample of Table 1 to become a part of the save the earth campaign of the entire mankind, with the hope that our studies will be found useful by the various governmental and non-governmental agencies and national and international organizations who are working today relentlessly to conserve the natural resources of this planet in their pristine form.

Table 6. A representative Analysis table for the DR-Hill Model of (2) for the arsenic contamination levels in the Shallow region as a function of that in the Deep (India Mark II) region for some of the representative values of the predictor

S. No.	Deep (India Mark II) (Arsenic contamination in $\times 10^2$ ppb) a	Slope of the curve of at this point , i.e., $f'(x)\|_{x=a}$	Area under the curve from $x = a$ up to this point, i.e., $\int_a^x f(x)\,dx$ in $(\times 10^2 \text{ppb})^2$	Total Arc length under the curve up to this point, i.e., $\int_a^x \sqrt{1+(f'(x))^2}dx$ in $\times 10^2$ppb
1.	4.1999999999999998E-01	5.71154235018e-05	0	0
2.	4.3600000000000000E-01	0.00603197769511	0.00819103804446	0.0160000317518
3.	4.5200000000000001E-01	0.518676265826	0.0163887859043	0.0322414775082
4.	4.6800000000000003E-01	6.41432650883	0.0249157577163	0.0942785915969
5.	4.8400000000000004E-01	0.243214207107	0.0345217833934	0.138154508328
6.	5.0000000000000000E-01	0.00402986533032	0.0443091907664	0.154211538004
7.	5.1600000000000001E-01	7.4637518388e-05	0.054100160794	0.170211554073
8.	5.3200000000000003E-01	1.5598633496e-06	0.0638911926543	0.186211554099
9.	5.4800000000000004E-01	3.60822483003e-08	0.0736822255475	0.202211554106
10.	5.6400000000000006E-01	2.77555756156e-09	0.0834732588343	0.218211554099
11.	5.8000000000000007E-01	0	0.0932642919472	0.2342115541

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