Engineering and Applied Science Letters

PSR Press

Article

Foundational concepts of circular intuitionistic fuzzy soft sets applied to sustainable decision-making for cement plant site selection

Fatima Zahra[1], Hafiz Inam ul Haq[1,*] and Muhammad Haroon[1]

[1] Department of Mathematics, University of Management and Technology Lahore, 54770 Pakistan
* Correspondence: sardarhafizinam@gmail.com

Received: 10 December 2024; Accepted: 27 February 2025; Published: 11 May 2025.

Abstract: The optimal selection of a site for cement plant development is a multifaceted decision-making process that demands careful consideration of environmental, economic, and social dimensions. This research delves into the utilization of Circular Intuitionistic Fuzzy Soft Sets (CIFSS) as an advanced mathematical framework to enhance the precision and reliability of sustainable decision-making in cement plant site selection. The CIFSS approach adeptly manages the inherent uncertainties and ambiguities associated with evaluating potential locations, offering a comprehensive methodology for assessing various criteria. By embedding CIFSS within the context of sustainable development, this technique provides decision-makers with a robust and adaptable tool for identifying the most appropriate site, thereby ensuring long-term viability and minimizing environmental impacts. The results underscore the effectiveness of CIFSS in facilitating complex, multi-criteria decision-making in industrial site selection, underscoring its broader applicability in sustainable infrastructure planning.

Keywords: fuzzy sets, intuitionistic fuzzy sets, circular intuitionistic fuzzy sets, soft sets, intuitionistic fuzzy soft sets

---

In the contemporary industrial sector, the process of selecting an optimal site for a cement plant is a decision of paramount importance, as it profoundly impacts the environmental sustainability, economic feasibility, and social acceptability of the project. The intricacies involved in this decision-making process are manifold, encompassing a wide range of factors such as environmental compliance [1], resource accessibility, logistical efficiency, community welfare, and long-term economic returns. Traditional decision-making methodologies often fall short in addressing the multifaceted and uncertain nature of these factors, leading to suboptimal site selections that could compromise the sustainability and success of the cement plant.To navigate the complexities inherent in such critical decisions [2], there has been a growing interest in the application of sophisticated mathematical models that can better manage the uncertainties and ambiguities associated with site selection. Among these advanced tools, Circular Intuitionistic Fuzzy Soft Sets (CIFSS) have emerged as a highly effective and versatile framework. CIFSS offers a structured yet flexible approach to decision-making, capable of accommodating a vast array of criteria and sub-criteria, each with varying degrees of importance and uncertainty.

This adaptability makes CIFSS particularly suitable for the nuanced task of cement plant site selection, where multiple conflicting factors must be balanced to achieve a sustainable outcome. The concept of Circular Intuitionistic Fuzzy Soft Sets is rooted in fuzzy set theory, which was developed to model the vagueness and imprecision inherent in real-world decision-making processes. Unlike traditional binary logic, which dictates that a statement is either true or false, fuzzy logic allows for degrees of truth, thereby providing a more accurate representation of the uncertainties involved in complex decisions. Intuitionistic fuzzy sets extend this concept further by introducing a degree of hesitation, reflecting the decision-maker's uncertainty about the degree of membership and non-membership of an element in a set. Circular intuitionistic fuzzy sets, as an extension, incorporate the notion of circularity, enabling a more refined analysis of cyclical or repetitive patterns in decision criteria, which are often encountered in environmental and economic assessments.The integration of CIFSS into the site selection process for cement plants represents a significant advancement in the field of sustainable industrial planning. By leveraging CIFSS, decision-makers are equipped with a powerful tool that can systematically evaluate potential sites across multiple dimensions, including environmental impact, economic viability, and social equity. This comprehensive approach ensures that all relevant factors are considered, and their interdependencies are adequately addressed, leading to more informed and balanced decisions.Moreover, the application of CIFSS in site selection aligns with the broader objectives of sustainable development. As the global industrial landscape continues to evolve, there is an increasing emphasis on ensuring that economic growth is achieved without compromising the health of the environment or the well-being of communities. Sustainable development requires that industrial projects, such as cement plants, are planned and executed in a manner that minimizes negative environmental impacts, optimizes resource use, and enhances social outcomes. CIFSS, with its ability to handle complex, multi-criteria decisions, provides a robust framework for achieving these objectives.

This paper delves into the foundational principles of Circular Intuitionistic Fuzzy Soft Sets and their practical application in the sustainable site selection process for cement plants. The discussion will elucidate how CIFSS can be employed to navigate the complexities of site evaluation, offering a more nuanced and comprehensive analysis than traditional methods. By incorporating CIFSS into the decision-making process, stakeholders are better positioned to make decisions that not only meet the economic requirements of the project but also fulfill the environmental and social responsibilities that are crucial for long-term sustainability. Furthermore, this paper will explore the broader implications of using CIFSS in industrial site selection, emphasizing its potential to enhance the overall decision-making process in other sectors where sustainability is a critical concern. The versatility of CIFSS makes it applicable not only to cement plant site selection but also to a wide range of industrial planning scenarios where complex, interrelated factors must be carefully balanced to achieve a sustainable outcome.

1. Aims and scope of study

The primary aim of this study is to develop and implement Circular Intuitionistic Fuzzy Soft Sets (CIFSS) as an advanced decision-making framework tailored specifically for the sustainable site selection of cement plants. Recognizing the intricate and multidimensional nature of site selection, this research endeavors to provide a method that can effectively navigate the complexities associated with balancing economic, environmental, and social considerations. Traditional decision-making approaches often struggle with the inherent uncertainty, ambiguity, and conflicting objectives that characterize such processes. By leveraging CIFSS, this study seeks to offer a more precise, reliable, and holistic approach that can accommodate a wide range of criteria, each with varying degrees of importance and uncertainty.The scope of this research extends beyond merely proposing a theoretical model; it involves a comprehensive exploration of the CIFSS framework, its theoretical underpinnings, and its practical application within the cement industry. The study begins with a thorough investigation of the conceptual foundation of CIFSS, examining its roots in fuzzy set theory and its evolution into a tool capable of addressing the cyclical and repetitive patterns often encountered in environmental and economic assessments. By integrating CIFSS into the site selection process, the research aims to create a robust framework that not only identifies the most viable locations for cement plants but also aligns with the overarching goals of sustainable development.To achieve this, the study includes the development of a tailored CIFSS-based framework that systematically evaluates potential sites across multiple dimensions, such as environmental impact, resource availability, logistical efficiency, and community acceptance. This framework is designed to address the unique challenges posed by the cement industry, particularly its high environmental impact and resource-intensive operations. The study also involves a comparative analysis between the CIFSS-based approach and traditional decision-making methods, highlighting the advantages of CIFSS in terms of accuracy, adaptability, and comprehensiveness.Furthermore, the research aims to provide actionable insights and practical recommendations for industry stakeholders, decision-makers, and policymakers. These recommendations will focus on the adoption and implementation of CIFSS in the site selection process, not only within the cement industry but also in other industrial sectors where sustainability is a critical concern. The study emphasizes the importance of aligning industrial site selection with broader environmental, economic, and social sustainability objectives, ensuring that decisions contribute to long-term viability and minimal environmental impact.In addition, the scope of this study includes empirical validation of the CIFSS framework through case studies or hypothetical scenarios. These examples will illustrate the practical benefits of using CIFSS for sustainable site selection, demonstrating how the framework can be applied to real-world situations. The research also explores the interdisciplinary integration of CIFSS, positioning it as a versatile tool that can enhance decision-making processes across various industries.Ultimately, this study seeks to establish CIFSS as a robust and reliable tool for optimizing industrial site selection in alignment with sustainable development goals. By providing a comprehensive, adaptable, and theoretically sound framework, this research aims to contribute to the advancement of sustainable industrial planning, ensuring that site selection processes are both effective and aligned with the broader objectives of economic growth, environmental protection, and social responsibility.

2. Literature review

The selection of an optimal site for industrial facilities, such as cement plants, is a critical decision that has significant implications for environmental sustainability, economic efficiency, and social acceptability. Over the years, various decision-making frameworks have been developed to assist in the complex process of site selection, with a particular focus on handling the uncertainties and multi-criteria nature of such decisions. Among these, fuzzy set theory and its extensions have gained prominence due to their ability to manage vagueness and imprecision in decision-making processes. Zadeh [1] introduced fuzzy set theory as a mathematical framework for dealing with uncertainty, where elements have degrees of membership rather than binary true or false values. This theory has been widely applied in multi-criteria decision-making (MCDM) to address the inherent uncertainties in evaluating multiple, often conflicting criteria. Over time, several extensions of fuzzy set theory have emerged, including intuitionistic fuzzy sets, which account for both membership and non-membership degrees along with a degree of hesitation [2]. These extensions provide a more nuanced approach to decision-making, particularly in scenarios where uncertainty plays a significant role.

Intuitionistic fuzzy sets have been increasingly utilized in decision-making processes that require a more sophisticated analysis of uncertainty. Their application spans various domains, including environmental management, supply chain optimization, and industrial site selection. For instance, intuitionistic fuzzy TOPSIS and AHP have been applied to rank alternatives in situations where decision-makers face ambiguity and conflicting criteria. These methods allow for a more comprehensive assessment by incorporating the hesitation degree, which reflects the uncertainty in the decision-making process.

Circular Intuitionistic Fuzzy Sets (CIFSS) an extension of intuitionistic fuzzy sets, incorporate the notion of circularity to address scenarios where cyclical or repetitive patterns influence decision-making criteria. CIFSS is particularly relevant in environmental and economic assessments, where factors such as resource availability, seasonal variations, and market fluctuations exhibit cyclic behavior. The concept of circularity enhances the ability to model these patterns more accurately, leading to better-informed decisions. However, the application of CIFSS in industrial site selection, specifically for cement plants, remains underexplored.

Sustainable site selection involves evaluating potential sites based on a wide range of criteria, including environmental impact, economic feasibility, and social responsibility. Traditional methods, such as AHP, TOPSIS, and GIS-based approaches, have been employed extensively in this context. While these methods are effective in certain scenarios, they often fall short in handling the complexity and uncertainty inherent in sustainable site selection. The integration of fuzzy set theory into these methods has shown promise in addressing these challenges by providing a more flexible and adaptive decision-making framework.

The application of CIFSS in sustainable decision-making is a relatively new area of research. CIFSS offers a structured approach to evaluating multiple criteria, each with varying degrees of uncertainty, in a cyclical context. This is particularly beneficial in the sustainable selection of cement plant sites, where factors such as environmental regulations, resource availability, and community acceptance are subject to cyclical variations. By incorporating CIFSS into the decision-making process, it is possible to achieve a more balanced and sustainable outcome that aligns with long-term environmental and economic goals. Despite the potential benefits, the application of CIFSS in industrial site selection, particularly for cement plants, is still in its infancy. Existing studies primarily focus on the theoretical aspects of CIFSS, with limited empirical validation in real-world scenarios. Furthermore, there is a lack of comprehensive frameworks that integrate CIFSS with other decision-making tools, such as GIS or AHP, to enhance the overall decision-making process. Future research should focus on developing and validating CIFSS-based frameworks for industrial site selection, with an emphasis on sustainability. Additionally, the exploration of hybrid approaches that combine CIFSS with other MCDM methods could offer a more robust solution to the complex challenges of sustainable site selection. The literature on fuzzy set theory and its extensions, particularly intuitionistic fuzzy sets, highlights their significance in addressing the uncertainties in decision-making processes. However, the application of Circular Intuitionistic Fuzzy Soft Sets in the sustainable selection of industrial sites, such as cement plants, represents a promising yet underexplored area. This study aims to fill this gap by developing a CIFSS-based framework for sustainable site selection, thereby contributing to the advancement of decision-making tools that support long-term sustainability goals.

3. Basic concepts

This section offers an overview of the fundamental definitions, the concepts of FCIFSS, and the historical background. To understand the key discoveries, it is essential to recall the development of soft sets as documented in the literature.

Definition 1 (Fuzzy Set [3]). A fuzzy set μ defined as $\mu = \{(x, \mu_X(x)) \mid x \in U\}$ such that $\mu_X : U \to [0,1]$ where $\mu_X(x)$ denotes the membership value of $x \in \mu$.

Below is the characteristics of fuzzy set.
If μ and ν are two fuzzy sets then for all $x \in U$,
(i) $\mu \cup \nu = \{(x, \max\{\mu_X(x), \nu_X(x)\})\}$,
(ii) $\mu \cap \nu = \{(x, \min\{\mu_X(x), \nu_X(x)\})\}$,
(iii) $\mu^c = \{(x, 1 - \mu_X(x)) \mid x \in U\}$.
Fuzzy sets emphasize the degree of truth when dealing with unclear scenarios, however, there are several circumstances where the degrees of falsity must be considered to properly apply fuzzy sets to such situations.

Definition 2 (Intuitionistic Fuzzy Set [4]). An intuitionistic fuzzy set Z is written as $Z = \{(y, \langle \alpha_Z(y), \beta_Z(y) \rangle) \mid y \in V\}$ such that $\alpha_Z : V \to [0,1]$ and $\beta_Z : V \to [0,1]$, where $\alpha_Z(y)$ and $\beta_Z(y)$ denote the truth value and falsity value of $y \in Z$ with $0 \leq \alpha_Z(y) + \beta_Z(y) \leq 1$ and the degree of hesitancy $\gamma_Z(y) = 1 - \alpha_Z(y) - \beta_Z(y)$.

Definition 3 (Soft Set [5]). A pair (Σ, σ) is defined as a soft set σ over N, where $\Sigma_N : T \to P(N)$.

Example 1. Let $N = \{p_1, p_2, p_3, p_4, p_5, p_6\}$, and $\Theta = \{v_1, v_2, v_3, v_4\}$. The approximate elements of the soft set $\sigma = (\Sigma, \Theta)$ are given by:

$$\Sigma(v_1) = \{p_1, p_3, p_6\},$$
$$\Sigma(v_2) = \{p_2, p_3, p_5\},$$
$$\Sigma(v_3) = \{p_4, p_5, p_6\},$$
$$\Sigma(v_4) = \{p_1, p_2, p_5\}.$$

The soft set σ is stated as

$$\sigma = \{\Sigma(v_1), \Sigma(v_2), \Sigma(v_3), \Sigma(v_4)\},$$

or

$$\sigma = \{(v_1, \{p_1, p_3, p_6\}), (v_2, \{p_2, p_3, p_5\}), (v_3, \{p_4, p_5, p_6\}), (v_4, \{p_1, p_2, p_5\})\}.$$

Definition 4 (Fuzzy Soft Set [6]). A pair (Φ_ξ, τ) is defined as a fuzzy soft set over N, where $\Phi_\xi : \tau \to P(\xi)$ and $P(\xi)$ is a collection of all fuzzy subsets over N, $\tau \subseteq Y$.

Definition 5 (Intuitionistic Fuzzy Soft Set [7]). A pair $(\Phi_{J\xi}, \tau)$ is referred to as an intuitionistic fuzzy soft set over N, where

$$\Phi_{J\xi} : \tau \to P(J\xi),$$

and $P(J\xi)$ is a collection of all IF subsets over N, $\tau \subseteq Y$.

Definition 6 (Circular Intuitionistic Fuzzy Set [8]). Let F be a fixed universe and B its subset. The set

$$B_s^* = \{(\hat{y}, \alpha_B(y), \beta_B(y); s) \mid y \in F\},$$

where $0 \leq \alpha_B(y) + \beta_B(y) \leq 1$ and $s \in [0,1]$ is a radius of the circle around each element $y \in F$, is called a Circular-IFS (C-IFS). The functions $\alpha_B : F \to [0,1]$ and $\beta_B : F \to [0,1]$ represent the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of an element $y \in F$ to the fixed set $B \subseteq F$.

Now, we can also define the function $\gamma_B : F \to [0,1]$ by

$$\gamma_B(y) = 1 - \alpha_B(y) - \beta_B(y),$$

which corresponds to the degree of indeterminacy (uncertainty, etc.).

4. Circular intuitionistic fuzzy soft set

Definition 7 (Circular Intuitionistic Fuzzy Soft Set). Consider N as a universal set and V as a set of parameters. Let $P(N)$ represent the set of all circular intuitionistic fuzzy sets of N. Let $S \subseteq V$. A pair (G, S) is defined as a circular intuitionistic fuzzy soft set over N, where G is a mapping given by $G : S \to P(N)$. The radius of each element in N varies within the range $[0,1]$, and can be mathematically represented as $t \in [0,1]$. Formally,

$$G : S \to CIF(N).$$

For conciseness, we will denote A_r^* as A_r whenever feasible.

Unlike traditional Intuitionistic Fuzzy Sets (IFS), where each element is depicted as a point within the intuitionistic fuzzy interpretation triangle, in this scenario, each element is represented by a circle centered at $(\mu_A(x), \nu_A(x))$ with a radius r.

The primary (secondary) geometric representation of the Circular-Intuitionistic Fuzzy Set (C-IFS) is shown in Figure 1.

It is clear that there are five distinct types of circles, as illustrated.

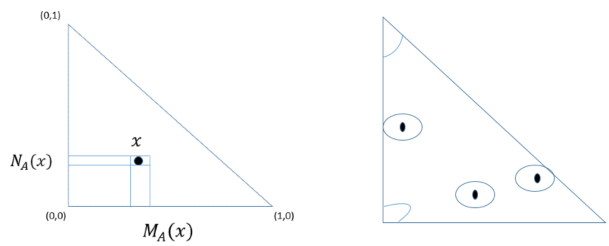

Figure 1. Geometric understanding of an element in an IVIFSS and circular IFSS

Definition 8 (Property of Two Circular Intuitionistic Fuzzy Soft Sets). For two circular intuitionistic fuzzy soft sets (H, T) and (J, U) over a common universe N, we say that (H, T) is an intuitionistic fuzzy soft subset of (J, U) if

(i) $T \subseteq U$

(ii) $\forall \delta \in T, H(\delta)$ is a circular intuitionistic fuzzy set of $J(\delta)$.

(iii) The radius of the first circular intuitionistic fuzzy soft set (H, T) is a subset of the radius of the second circular intuitionistic fuzzy soft set (J, U) over a common universe N, and satisfies the condition $s \in [0,1]$.

Definition 9 (Equality of Two Circular Intuitionistic Fuzzy Soft Sets). Two circular intuitionistic fuzzy soft sets (H, T) and (J, U) over a common universe N are said to be fuzzy soft equal if (H, T) is a circular intuitionistic fuzzy soft subset of (J, U) and (J, U) is a circular intuitionistic fuzzy soft subset of (H, T).

Definition 10. Let $\eta = (H, T)$ be a circular intuitionistic fuzzy soft set over N, where $T \subseteq W$ and W is a set of parameters. For $m, n \in [0, 1]$, the (m, n)-level soft set of η is a crisp soft set $L(\eta; m, n) = (H_{(m,n)}, T)$ defined by

$$H_{(m,n)}(\delta) = L(H(\delta); m, n) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m \text{ and } \beta_{(H(\delta))}(y) \leq n\} \quad \text{for all } \delta \in T.$$

Theorem 1. *Let $\eta = (H, S)$ be a circular intuitionistic fuzzy soft set over N, where $S \subseteq V$ and V is a set of parameters. $L(\eta; m_1, n_1)$ and $L(\eta; m_2, n_2)$ are (m_1, n_1)-level soft set and (m_2, n_2)-level soft set of η, respectively, where $m_1, n_1, m_2, n_2 \in [0, 1]$. If $m_2 \leq m_1$ and $n_2 \leq n_1$, then we have $L(\eta; m_1, n_1) \subseteq L(\eta; m_2, n_2)$.*

Proof. Let $L(\eta; m_1, n_1) = \{(H_{(m_1,n_1)}, S)\}$, where

$$H_{(m_1,n_1)}(\delta) = L(H(\delta); m_1, n_1) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_1 \text{ and } \beta_{(H(\delta))}(y) \leq n_1\},$$

for all $\delta \in S$. Let $L(\eta; m_2, n_2) = \{(H_{(m_2,n_2)}, S)\}$, where

$$H_{(m_2,n_2)}(\delta) = L(H(\delta); m_2, n_2) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_2 \text{ and } \beta_{(H(\delta))}(y) \leq n_2\},$$

for all $\delta \in S$. Obviously, $S \subseteq S$. We shall show that $\forall \delta \in S$, $H_{(m_1,n_1)}(\delta) \subseteq H_{(m_2,n_2)}(\delta)$.

Since $m_2 \leq m_1$ and $n_2 \leq n_1$, for all $\delta \in S$, we have the following:

$$\{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_1 \text{ and } \beta_{(H(\delta))}(y) \leq n_1\} \subseteq \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_2 \text{ and } \beta_{(H(\delta))}(y) \leq n_2\}.$$

Since $H_{(m_1,n_1)}(\delta) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_1 \text{ and } \beta_{(H(\delta))}(y) \leq n_1\}$ and $H_{(m_2,n_2)}(\delta) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m_2 \text{ and } \beta_{(H(\delta))}(y) \leq n_2\}$, we have $H_{(m_1,n_1)}(\delta) \subseteq H_{(m_2,n_2)}(\delta)$. □

Theorem 2. *Let $\eta = (H, S)$ and $\theta = (J, S)$ be two circular intuitionistic fuzzy soft sets over N, where $S \subseteq V$ and V is a set of parameters. $L(\eta; m, n)$ and $L(\theta; m, n)$ are (m, n)-level soft sets of η and θ respectively, where $m, n \in [0, 1]$. If $\eta \subseteq \theta$, then we have $L(\eta; m, n) \subseteq L(\theta; m, n)$.*

Proof. Let $L(\eta; m, n) = (H_{(m,n)}, S)$, where

$$H_{(m,n)}(\delta) = L(H(\delta); m, n) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m \text{ and } \beta_{(H(\delta))}(y) \leq n\},$$

for all $\delta \in S$. Let $L(\theta; m, n) = (J_{(m,n)}, S)$, where

$$J_{(m,n)}(\delta) = L(J(\delta); m, n) = \{y \in N \mid \alpha_{(J(\delta))}(y) \geq m \text{ and } \beta_{(J(\delta))}(y) \leq n\},$$

for all $\delta \in S$. Obviously, $S \subseteq S$. We shall show that $\forall \delta \in S$, $H_{(m,n)}(\delta) \subseteq J_{(m,n)}(\delta)$.

Since $\eta \subseteq \theta$, we have the following:

$$\alpha_{H(\delta)}(y) \leq \alpha_{J(\delta)}(y) \quad \text{and} \quad \beta_{H(\delta)}(y) \geq \beta_{J(\delta)}(y) \quad \text{for all } y \in N \text{ and } \delta \in S.$$

Assume that $y \in H_{(m,n)}(\delta)$. Since $H_{(m,n)}(\delta) = \{y \in N \mid \alpha_{(H(\delta))}(y) \geq m \text{ and } \beta_{(H(\delta))}(y) \leq n\}$, we have that $\alpha_{(H(\delta))}(y) \geq m$ and $\beta_{(H(\delta))}(y) \leq n$. Since $\alpha_{H(\delta)}(y) \leq \alpha_{J(\delta)}(y)$ and $\beta_{H(\delta)}(y) \geq \beta_{J(\delta)}(y)$, we have $\alpha_{(J(\delta))}(y) \geq m$ and $\beta_{(J(\delta))}(y) \leq n$.

Hence, $y \in \{y \in N \mid \alpha_{(J(\delta))}(y) \geq m \text{ and } \beta_{(J(\delta))}(y) \leq n\}$. Since $J_{(m,n)}(\delta) = \{y \in N \mid \alpha_{(J(\delta))}(y) \geq m \text{ and } \beta_{(J(\delta))}(y) \leq n\}$, we have $y \in J_{(m,n)}(\delta)$. Thus, we have that $H_{(m,n)}(\delta) \subseteq J_{(m,n)}(\delta)$.

Consequently, $L(\eta; m, n) \subseteq L(\theta; m, n)$. □

Definition 11. Let $\eta = (H, S)$ be a circular intuitionistic fuzzy soft set over N, where $S \subseteq V$ and V is a set of parameters. Let $\kappa : S \to [0, 1] \times [0, 1]$ be a circular intuitionistic fuzzy set in S which is called a threshold circular intuitionistic fuzzy set. The level soft set of η with respect to κ is a crisp soft set $L(\eta; \kappa) = (H, S)$ defined by

$$H_\kappa(\delta) = L(H(\delta); \kappa(\delta)) = \{y \in N \mid \alpha_{H(\delta)}(y) \geq \alpha_{\kappa(\delta)}(y) \text{ and } \gamma_{\kappa(\delta)}(y) \leq \gamma_{\kappa(\delta)}(y)\} \text{ for all } \delta \in S.$$

Obviously, the level soft sets of circular intuitionistic fuzzy soft sets with respect to a circular intuitionistic fuzzy set are extensions of the level soft sets of fuzzy soft sets with respect to a fuzzy set.

Theorem 3. *Let $\eta = (H, S)$ be a circular intuitionistic fuzzy soft set over N, where $S \subseteq V$ and V is a set of parameters. Let $\kappa_1 : S \to [0,1] \times [0,1]$ and $\kappa_2 : S \to [0,1] \times [0,1]$ be two threshold circular intuitionistic fuzzy sets. $L(\eta; \kappa_1) = (H_{\kappa_1}, S)$ and $L(\eta; \kappa_2) = (J_{\kappa_2}, S)$ are the level soft sets of η with respect to κ_1 and κ_2, respectively. If*

$$\alpha_{\kappa_2(\delta)}(y) \le \alpha_{\kappa_1(\delta)}(y) \text{ and } \gamma_{\kappa_2(\delta)}(y) \ge \gamma_{\kappa_1(\delta)}(y) \text{ for all } \delta \in S,$$

then we have $L(\eta; \kappa_1) \subset L(\eta; \kappa_2)$.

Proof. The proof is similar to Theorem 1. □

Theorem 4. *Let $\eta = (H, S)$ and $\theta = (J, S)$ be two circular intuitionistic fuzzy soft sets over N, where $S \subseteq V$ and V is a set of parameters. Let $\kappa : S \to [0,1] \times [0,1]$ be a threshold circular intuitionistic fuzzy set. $L(\eta; \kappa) = (H_\kappa, S)$ and $L(\theta; \kappa) = (J_\kappa, S)$ are the level soft sets of η and θ with respect to κ, respectively. If $\eta \subset \theta$, then we have $L(\eta; \kappa) \subset L(\theta; \kappa)$.*

Proof. The proof is similar to Theorem 2. □

5. Circular intuitionistic fuzzy soft sets and its applications multi decision making for cement plant site selection

Example 2 (evaluating environmental impact of cement plant sites)**.** Consider three potential sites $S = \{s_1, s_2, s_3\}$ for a new cement plant based on the criterion of Environmental Impact (c_1). Using Circular Intuitionistic Fuzzy Soft Sets (CIFSS), the decision-makers assign the following intuitionistic fuzzy values:

- For site s_1:
  - Membership value $\mu_{s_1,c_1} = 0.75$
  - Non-membership value $\nu_{s_1,c_1} = 0.15$
  - Hesitation degree $\pi_{s_1,c_1} = 0.1$
- For site s_2:
  - Membership value $\mu_{s_2,c_1} = 0.60$
  - Non-membership value $\nu_{s_2,c_1} = 0.25$
  - Hesitation degree $\pi_{s_2,c_1} = 0.15$
- For site s_3:
  - Membership value $\mu_{s_3,c_1} = 0.80$
  - Non-membership value $\nu_{s_3,c_1} = 0.10$
  - Hesitation degree $\pi_{s_3,c_1} = 0.10$

The overall evaluation for each site regarding environmental impact is given by:

$$E(s_i) = \mu_{s_i,c_1} - \nu_{s_i,c_1} - \pi_{s_i,c_1}.$$

Calculating the values:

$$E(s_1) = 0.75 - 0.15 - 0.1 = 0.50,$$

$$E(s_2) = 0.60 - 0.25 - 0.15 = 0.20,$$

$$E(s_3) = 0.80 - 0.10 - 0.10 = 0.60.$$

Based on environmental impact, site s_3 is the most favorable option, as it has the highest overall evaluation score.

Lemma 1 (Effect of Minimizing Hesitation in Environmental Decisions)**.** *For any site s_i evaluated under the criterion of Environmental Impact (c_1), minimizing the hesitation degree π_{s_i,c_1} results in a higher overall evaluation $E(s_i)$, provided the membership (μ_{s_i,c_1}) and non-membership (ν_{s_i,c_1}) values are constant.*

Proof. The overall evaluation $E(s_i)$ for site s_i under criterion c_1 is expressed as:

$$E(s_i) = \mu_{s_i,c_1} - \nu_{s_i,c_1} - \pi_{s_i,c_1}.$$

Given that μ_{s_i,c_1} and ν_{s_i,c_1} are constants, $E(s_i)$ is inversely related to the hesitation degree π_{s_i,c_1}. Hence, reducing π_{s_i,c_1} increases $E(s_i)$, maximizing the site's favorability regarding environmental impact. □

Example 3 (Evaluating Economic Feasibility of Cement Plant Sites)**.** Consider the same three potential sites $S = \{s_1, s_2, s_3\}$ for a new cement plant, now evaluated based on Economic Feasibility (c_2). The CIFSS values are as follows:

- For site s_1:
  - Membership value $\mu_{s_1,c_2} = 0.85$
  - Non-membership value $\nu_{s_1,c_2} = 0.10$
  - Hesitation degree $\pi_{s_1,c_2} = 0.05$
- For site s_2:
  - Membership value $\mu_{s_2,c_2} = 0.70$
  - Non-membership value $\nu_{s_2,c_2} = 0.20$
  - Hesitation degree $\pi_{s_2,c_2} = 0.10$
- For site s_3:
  - Membership value $\mu_{s_3,c_2} = 0.65$
  - Non-membership value $\nu_{s_3,c_2} = 0.25$
  - Hesitation degree $\pi_{s_3,c_2} = 0.10$

The overall evaluation for each site in terms of economic feasibility is:

$$E(s_i) = \mu_{s_i,c_2} - \nu_{s_i,c_2} - \pi_{s_i,c_2}.$$

Calculating the values:

$$E(s_1) = 0.85 - 0.10 - 0.05 = 0.70,$$

$$E(s_2) = 0.70 - 0.20 - 0.10 = 0.40,$$

$$E(s_3) = 0.65 - 0.25 - 0.10 = 0.30.$$

In terms of economic feasibility, site s_1 is the most favorable option.

Lemma 2 (Balancing environmental impact and economic feasibility)**.** *When evaluating potential sites for a cement plant under conflicting criteria such as Environmental Impact (c_1) and Economic Feasibility (c_2), Circular Intuitionistic Fuzzy Soft Sets (CIFSS) allow decision-makers to balance these criteria by adjusting the weights assigned to each and incorporating hesitation degrees.*

Proof. The overall evaluation $E(s_i)$ considering both Environmental Impact (c_1) and Economic Feasibility (c_2) is given by:

$$E(s_i) = w_1(\mu_{s_i,c_1} - \nu_{s_i,c_1} - \pi_{s_i,c_1}) + w_2(\mu_{s_i,c_2} - \nu_{s_i,c_2} - \pi_{s_i,c_2}),$$

where w_1 and w_2 are weights representing the relative importance of Environmental Impact and Economic Feasibility, respectively.

By adjusting these weights w_1 and w_2 based on the priorities of the decision-makers, and considering the hesitation degrees π_{s_i,c_1} and π_{s_i,c_2}, CIFSS facilitates a balanced and informed decision-making process that accounts for both environmental sustainability and economic viability. □

Example 4 (Evaluating Community Acceptance for Cement Plant Sites)**.** Consider evaluating the same three sites $S = \{s_1, s_2, s_3\}$ based on Community Acceptance (c_3). The CIFSS values are:

- For site s_1:
  - Membership value $\mu_{s_1,c_3} = 0.70$
  - Non-membership value $\nu_{s_1,c_3} = 0.20$
  - Hesitation degree $\pi_{s_1,c_3} = 0.10$
- For site s_2:
  - Membership value $\mu_{s_2,c_3} = 0.65$
  - Non-membership value $\nu_{s_2,c_3} = 0.25$
  - Hesitation degree $\pi_{s_2,c_3} = 0.10$
- For site s_3:
  - Membership value $\mu_{s_3,c_3} = 0.75$
  - Non-membership value $\nu_{s_3,c_3} = 0.15$
  - Hesitation degree $\pi_{s_3,c_3} = 0.10$

The overall evaluation for each site concerning community acceptance is:

$$E(s_i) = \mu_{s_i,c_3} - \nu_{s_i,c_3} - \pi_{s_i,c_3}.$$

Calculating the values:

$$E(s_1) = 0.70 - 0.20 - 0.10 = 0.40,$$

$$E(s_2) = 0.65 - 0.25 - 0.10 = 0.30,$$

$$E(s_3) = 0.75 - 0.15 - 0.10 = 0.50.$$

In terms of community acceptance, site s_3 is the most favorable option, as it has the highest overall evaluation score.

Lemma 3 (Influence of Hesitation Degree on Multi-Criteria Decision-Making)**.** *In a multi-criteria decision-making process involving Environmental Impact (c_1), Economic Feasibility (c_2), and Community Acceptance (c_3), the hesitation degrees π_{s_i,c_j} associated with each criterion have a significant impact on the final decision. Minimizing these hesitation degrees across all criteria leads to a more robust and confident selection of the optimal site.*

Proof. Let the overall evaluation $E(s_i)$ of site s_i considering all three criteria be represented as:

$$E(s_i) = w_1(\mu_{s_i,c_1} - \nu_{s_i,c_1} - \pi_{s_i,c_1}) + w_2(\mu_{s_i,c_2} - \nu_{s_i,c_2} - \pi_{s_i,c_2}) + w_3(\mu_{s_i,c_3} - \nu_{s_i,c_3} - \pi_{s_i,c_3}),$$

where w_1, w_2, and w_3 are weights corresponding to the importance of Environmental Impact, Economic Feasibility, and Community Acceptance, respectively.

Given that μ_{s_i,c_j} and ν_{s_i,c_j} are constants, the overall evaluation $E(s_i)$ is directly affected by the hesitation degrees π_{s_i,c_j} for each criterion. By minimizing π_{s_i,c_1}, π_{s_i,c_2}, and π_{s_i,c_3}, the overall evaluation $E(s_i)$ increases, leading to a more reliable and higher confidence decision for the optimal site selection. □

6. Environmental, social, and economic impact assessment (escia) of a cement plant

Environmental, social, and economic impact assessment is a crucial component in the decision-making process for siting cement plants. Let X represent the set of all possible impacts associated with a cement plant, categorized into environmental (E), social (S), and economic (C) impacts. Each category can be further broken down into specific impact factors.

6.1. Mathematical representation

Let $x_i \in X$ represent an individual impact factor, and consider the following fuzzy soft set representation:

$$\text{ESCIA}(s_i) = \{(x_i, (\mu(x_i), \nu(x_i), \pi(x_i))) \mid x_i \in X\},$$

where $\mu(x_i)$ is the membership degree, $\nu(x_i)$ is the non-membership degree, and $\pi(x_i)$ is the hesitation degree associated with impact x_i.

For each impact category:

- Environmental Impact (E): This includes greenhouse gas emissions (e.g., CO_2), air emissions (e.g., NO_x, SO_2), dust and particulate emissions, noise, vibration, and visual impacts.

$$E(s_i) = \sum_{j=1}^{n} (\mu(e_j) - \nu(e_j) - \pi(e_j)) \quad \text{where } e_j \in E.$$

- Social Impact (S): This involves direct and indirect impacts on local communities, including health risks, displacement, and changes in quality of life.

$$S(s_i) = \sum_{k=1}^{m} (\mu(s_k) - \nu(s_k) - \pi(s_k)) \quad \text{where } s_k \in S.$$

- Economic Impact (C): Economic impacts include job creation, effects on local businesses, and overall contribution to the economy.

$$C(s_i) = \sum_{l=1}^{p} (\mu(c_l) - \nu(c_l) - \pi(c_l)) \quad \text{where } c_l \in C.$$

The overall impact of the cement plant can then be represented as the aggregate impact:

$$\text{Total Impact}(s_i) = w_1 E(s_i) + w_2 S(s_i) + w_3 C(s_i),$$

where w_1, w_2, and w_3 are the weights assigned to environmental, social, and economic impacts, respectively, based on the priorities of the decision-makers.

6.2. Application of CIFSS

The Circular Intuitionistic Fuzzy Soft Set (CIFSS) framework allows decision-makers to evaluate each impact factor with the corresponding membership, non-membership, and hesitation degrees, thereby providing a more nuanced analysis. The CIFSS approach can help to:

- Identify major negative and positive impacts.
- Customize the siting decision process.
- Employ a prevention strategy over mitigation or compensation strategies.

For example, if e_1 represents CO_2 emissions:

$$\text{Impact of } CO_2(s_i) = (\mu(e_1) - \nu(e_1) - \pi(e_1)) \quad \text{where } e_1 \in E.$$

A higher value of $\mu(e_1)$ with a lower $\nu(e_1)$ and $\pi(e_1)$ would indicate a significant environmental impact, thus necessitating priority in addressing this factor during decision-making.

7. Environmental impacts of cement plants with CIFSS integration

The emission of carbon dioxide (CO_2) is a significant environmental challenge associated with cement plants. Approximately 50% of total CO_2 emissions from a cement plant are attributable to the calcination process, where limestone ($CaCO_3$) is converted to lime (CaO) and carbon dioxide (CO_2). This process is primarily driven by the combustion of fossil fuels during various stages, such as manufacturing operations (40%), transportation (5%), and on-site electricity generation (5%).

The sources of dust emissions are mainly linked to the operation of kilns, raw mills, clinker coolers, and cement mills. In particular, noise pollution arises from blasting or drilling activities associated with quarrying and the grinding of cement.

Figure 2. Enviromental, social and economically impact of cement plant

7.1. Mathematical formulation using CIFSS

Let $X = \{x_1, x_2, \ldots, x_n\}$ represent the set of environmental impact factors related to a cement plant. These factors include CO_2 emissions, dust emissions, noise pollution, and the impacts of transportation. For each impact factor $x_i \in X$, the Circular Intuitionistic Fuzzy Soft Set (CIFSS) approach assigns a membership degree $\mu(x_i)$, a non-membership degree $\nu(x_i)$, and a hesitation degree $\pi(x_i)$ as follows:

$$\text{Impact}(x_i) = (\mu(x_i) - \nu(x_i) - \pi(x_i)).$$

For example, the impact of CO_2 emissions can be represented as:

$$\text{Impact}(CO_2) = \mu(CO_2) - \nu(CO_2) - \pi(CO_2),$$

where $\mu(CO_2)$ is the membership degree indicating the extent to which CO_2 emissions contribute to environmental degradation, $\nu(CO_2)$ represents the degree to which CO_2 emissions do not contribute (e.g., due to mitigation measures), and $\pi(CO_2)$ captures the uncertainty or hesitation in the evaluation.

7.2. Traffic and quarrying impacts

Cement plants also impose substantial traffic impacts, particularly due to the transportation of raw materials and finished products. These activities contribute to soil contamination, noise pollution, vibrations, and dust, all of which present significant health and safety hazards. Additionally, the environmental concerns associated with quarrying operations include dust pollution, noise, vibrations, and detrimental effects on land use and biodiversity.

Mathematically, the impact of traffic and quarrying activities can be modeled using CIFSS as:

$$\text{Impact}(x_i) = \sum_{i=1}^{m} (\mu(x_i) - \nu(x_i) - \pi(x_i)) \quad \text{for } x_i \in \{\text{traffic, quarrying}\}.$$

Here, each impact factor x_i is assessed for its contribution to environmental and social risks, and the CIFSS approach allows for a more refined evaluation by considering the uncertainty ($\pi(x_i)$) inherent in such assessments.

8. Impact of cement plants on biodiversity and karst landscapes with CIFSS integration

Cement plants exert considerable pressure on biodiversity, particularly in areas where quarrying and cement production occur. These impacts manifest in several ways:

1. Displacement of existing ecosystems, including flora and fauna, in regions earmarked for quarrying and plant construction.
2. Degradation and disruption of biodiversity in the vicinity of operational activities.
3. Creation of land uses that conflict with adjacent landscapes, thereby disturbing neighboring ecosystems.

8.1. Mathematical representation Using CIFSS

Let $Y = \{y_1, y_2, \ldots, y_n\}$ represent the set of biodiversity impact factors. Each factor $y_i \in Y$ can be evaluated using the Circular Intuitionistic Fuzzy Soft Set (CIFSS) framework, which assigns a membership degree $\mu(y_i)$, a non-membership degree $\nu(y_i)$, and a hesitation degree $\pi(y_i)$. The impact of a factor y_i on biodiversity is mathematically expressed as:

$$\text{Biodiversity Impact}(y_i) = \mu(y_i) - \nu(y_i) - \pi(y_i).$$

For example, the displacement of ecosystems y_1 can be quantified as:

$$\text{Impact}(y_1) = \mu(y_1) - \nu(y_1) - \pi(y_1),$$

where $\mu(y_1)$ measures the extent to which displacement affects biodiversity, $\nu(y_1)$ reflects the degree of non-impact (e.g., through conservation efforts), and $\pi(y_1)$ captures the uncertainty in this evaluation.

8.2. Impact on karst areas

Cement plants are frequently sited in karst regions, which are particularly sensitive due to their unique geological characteristics. Karsts are geological formations resulting from the dissolution of carbonate minerals found in limestone and other sedimentary rocks. The following attributes make karst areas valuable yet vulnerable:

- Rich, fertile soils that are often used for agriculture.
- Cultural and historical significance due to the preservation of past environmental and historical data.

The threat posed by cement plants in karst landscapes can be evaluated using CIFSS as follows:

$$\text{Karst Impact}(z_i) = \sum_{i=1}^{m} (\mu(z_i) - \nu(z_i) - \pi(z_i)) \quad \text{for } z_i \in \{\text{soil degradation, biodiversity loss, cultural impact}\}.$$

Here, z_i represents the specific impact factors on karst landscapes, with CIFSS allowing for a detailed assessment of the negative effects on these sensitive areas.

9. Conclusion and future work

This study has investigated the application of Circular Intuitionistic Fuzzy Soft Sets (CIFSS) for sustainable site selection in the cement industry, demonstrating its effectiveness in addressing the complexities and uncertainties inherent in multi-criteria decision-making. The CIFSS framework provides a structured yet flexible approach, enabling decision-makers to balance environmental, economic, and social considerations in a more comprehensive and sustainable manner.

9.1. Future work

Future research should aim to empirically validate the CIFSS framework in real-world site selection cases, enhancing its practical applicability. Additionally, the integration of CIFSS with other decision-making tools such as Geographic Information Systems (GIS) and Analytic Hierarchy Process (AHP) could further refine its utility. Expanding the application of CIFSS to other industries with similar sustainability challenges and developing accessible decision-support software will be key steps in broadening the framework's impact.

Acknowledgments: No specific funding from government, industry, or non-benefit subsidizing organizations was provided for this study.

Conflicts of Interest: The authors have reported no conflicts of interest.

References

[1] Zadeh, L. (1965). Fuzzy sets. *Information and Control*, *8*, 338–353. Elsevier.
[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96.
[3] Zadeh, L. A., Klir, G. J., & Yuan, B. (1996). *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected papers* (Vol. 6). World Scientific.
[4] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96.
[5] Zadeh, L. A., Klir, G. J., & Yuan, B. (1996). *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected papers* (Vol. 6). World Scientific.
[6] Majumdar, P., & Samanta, S. K. (2010). Generalised fuzzy soft sets. *Computers & Mathematics with Applications*, *59*(4), 1425–1432.
[7] Agarwal, M., Biswas, K. K., & Hanmandlu, M. (2013). Generalized intuitionistic fuzzy soft sets with applications in decision-making. *Applied Soft Computing*, *13*(8), 3552–3566.
[8] Atanassov, K. T. (2020). Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *39*(5), 5981–5986.