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Review of microscopic traffic flow models: Historical developments and future trends

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Abstract: Traffic congestion presents a critical challenge in contemporary urban environments, necessitating the development of effective traffic management systems. Microscopic traffic flow models, which offer detailed insights into individual vehicle dynamics such as car-following and lane-changing behaviors, are pivotal in addressing these challenges. However, a comprehensive review synthesizing the advancements and research trends in this field has been lacking. This paper presents a systematic review of major microscopic traffic flow research from 1950 to 2023. Our extensive search across multiple academic databases identifies significant methodologies and model equations, highlighting notable advancements in the field. The presentation reveals critical trends, including the integration of connected and autonomous vehicles, the application of machine learning techniques, and the increasing reliance on real-time data for traffic management. This paper provides a foundation for future research directions and contributes to the ongoing development of more efficient and sustainable urban traffic management strategies.

Keywords: microscopic traffic flow models, car-following behavior, lane-changing behavior, traffic optimization, connected vehicles, urban traffic management

1. Introduction

Efficient traffic management is fundamental to contemporary urban planning and infrastructure development. As urban populations continue to grow and the pace of urbanization accelerates, traffic congestion has emerged as a pervasive challenge that affects vehicle flow, public safety, environmental sustainability, and economic productivity. Cities around the globe face mounting pressures related to high traffic volumes, safety concerns, and the demand for more effective transport systems. These challenges emphasize the necessity of understanding the complex interactions between individual vehicles and the broader traffic ecosystem.

Traditional macroscopic traffic models have long been employed to analyze traffic flow by treating it as a continuous stream, akin to fluid dynamics. While these models have provided valuable insights into overarching traffic trends and facilitated large-scale infrastructure planning, they often fall short in capturing the intricacies of individual vehicle behaviors, including acceleration, deceleration, lane-changing, and vehicle-following dynamics. These finer details are critical for addressing daily issues related to traffic congestion, optimizing traffic flow, and enhancing road safety. Furthermore, with the advent of new technologies, such as connected vehicles and autonomous driving systems, a deeper understanding of individual vehicle interactions has become increasingly essential.

Microscopic traffic flow models emerged as a vital tool in traffic engineering. Unlike their macroscopic counterparts, microscopic models concentrate on the behavior of individual vehicles and drivers, providing a granular perspective on traffic dynamics. These models simulate the decision-making processes of drivers in real-time, taking into account factors such as how they follow other vehicles, change lanes, and adjust their speeds based on traffic conditions. The capacity of microscopic models to capture the complexity of driver behavior renders them invaluable for developing intelligent transportation systems (ITS), designed to optimize traffic flow, alleviate congestion, and improve safety through the integration of advanced technologies and real-time data.

Despite the substantial body of research on microscopic traffic models, there exists a notable gap in the literature regarding comprehensive reviews that synthesize their development and trends within this domain. Previous reviews often focus narrowly on specific models or aspects, neglecting the broader context and the interrelationships among different modeling approaches. The rapid proliferation of studies in recent years has made it increasingly challenging for researchers and practitioners to stay abreast of key advancements and methodologies. As traffic modeling continues to evolve with the integration of new technologies—such as autonomous vehicles, machine learning algorithms, and real-time data analysis—it is imperative to understand the historical progression and emerging trends in microscopic traffic flow models.

This paper seeks to address this gap by providing a thorough review of microscopic traffic flow models, with a particular emphasis on their historical evolution from 1950 to 2023. This study aims to highlight significant methodologies and model equations that have shaped the development of these models. Furthermore, the paper identifies critical trends, including the integration of connected and autonomous vehicles, the application of machine learning techniques, and the increasing reliance on real-time data for traffic management. These findings are intended to inform future research directions and contribute to the ongoing development of more efficient and sustainable urban traffic management strategies.

2. Some historical micro developments

The evolution of traffic flow models has played a crucial role in advancing vehicular flow. Beginning with simple yet impactful theoretical frameworks, early models established the foundation for much of the work that followed in traffic analysis. Over the decades, these models have become increasingly sophisticated, incorporating more variables and refined mathematical formulations to capture the complex interactions between vehicles and drivers. This section explores key developments in traffic flow modeling, starting from the foundational work in vehicle-following behavior.

One of the earliest breakthroughs in this field was the work by [1], who introduced a theoretical framework to model how drivers maintain safe following distances. His model defined the desired following distance, $D = L + TV$, where L is the lead vehicle length, T the reaction time or headway, and V the speed of the following vehicle. This simple yet effective model laid the foundation for subsequent models such as [2], which explored three-dimensional systems using spin interaction theory. Along these lines, [3] extended this modeling approach by developing a dynamic system for vehicle-following interactions, where acceleration $a_n(t) = \alpha(v_{n-1}(t) - v_n(t))$, with α representing a sensitivity constant, to describe the behavior of vehicles. Further expanding these ideas, [4] introduced a model incorporating both speed and relative velocity adjustments, described by

$$D_n(t) = \phi(V_n(t)) + \psi(V_{n-1}(t) - V_n(t)),$$

where $D_n(t)$ represents the distance between vehicles, while the stability of traffic flow using a model similar to Chandler's was analyzed, focusing on how velocity disturbances propagate and lead to potential traffic jams [5].

Subsequent models further refined the understanding of traffic behavior. These include [6], a statistical mechanics approach to traffic flow dynamics accounting for the transition from free flow to congestion; [7], a follow-the-leader model for single-lane traffic incorporating vehicle concentration and jam density; and [8], a vehicle-following model incorporating reaction time R_T , maximum acceleration $a_{\max}$, and desired speed V_d , expressed as

$$u_n(t + R_T) = \min \left(u_n(t) + 2.5a_{\max}\tau \left(1 - \frac{u_n(t)}{V_d} \right) \left(0.025 + \frac{u_n(t)}{V_d} \right), \text{braking constraint} \right).$$

This concept of external forces was extended to traffic dynamics by [9] with the Rondo approach, which explains the formation of traffic patterns. The model

$$F(t + T) = F(t) - v_B T + V_{opt}[f(t)],$$

incorporates a delay parameter T , backward velocity v_B , and an optimal velocity function to capture systematic traffic behavior. This framework is applicable to traffic management, as explored by [10], who focused on

optimizing green time and queue management. Expanding on these concepts, [11] used the asymmetric exclusion process to study systems such as traffic flow and ion transport, modeled by

$$\frac{dP}{dt} = \sum_{i=1}^{L-1} [w_{i,i+1}P_{i+1} - w_{i+1,i}P_i],$$

with P representing the probability distribution and $w_{i,i+1}$ the transition rates between lattice sites. Afterward, [12] contributed to traffic flow analysis with the optimal velocity model,

$$V_d = v_0 \tanh\left(\frac{s_i - s_0}{\Delta s}\right) + \tanh\left(\frac{s_0}{\Delta s}\right),$$

focusing on driver response delays and how headway s_i influences desired velocity V_d . [13] extended these ideas to urban traffic, focusing on stop-and-go phenomena, with a model

$$\frac{dV}{dt} = V^*(s, V, V'_m) - V,$$

that accounts for the relative velocity V'_m between vehicles. Finally, [14] analyzed single-vehicle data using models such as

$$V_n \rightarrow \min(V_{n+1}, V_{\max}), \quad X_n \rightarrow X_n + V_n,$$

exploring microscopic structures in traffic states.

This subsequent work [15] applied kinetic theory to vehicular traffic, proposing a model for vehicle acceleration

$$a(t) = \frac{1}{\tau} [V_{\text{opt}}(\Delta x_n(t)) - V_n(t)],$$

exploring optimal headway and velocity dynamics in traffic systems. In the same year, [16] investigated the formation of congestion patterns and various traffic states on highways, employing the intelligent driver model

$$\frac{\delta V}{\delta t} = a_{\max} \left(1 - \left(\frac{V}{V_d} \right)^4 - \left(\frac{s^*}{h} \right)^2 \right),$$

to capture the interplay between a vehicle's velocity V , desired velocity V_d , safety distance s^* , and actual distance to the leading vehicle h . [17] explored traffic flow using queuing theory, focusing on $M/M/1$, $M/G/1$, and $G/G/1$ models, providing insight into traffic intensity and time spent in the system. Finally, [18] aimed to understand lane-changing behavior on urban streets, introducing models for speed differences such as

$$SD = V_T - V_H, \quad SA = V_{Ld} - V_H,$$

where V_T is the desired speed of the target vehicle and V_H is the speed of the head vehicle.

In 2001, research focused on improving stop-and-go Advanced Driver Assistance Systems (ADAS) led to the development of a model for low-speed distance-keeping between vehicles [19]. The model is expressed as

$$a = k_1 \cdot v_r + k_2 \cdot (DX - DX_{\text{des}}),$$

where a is the acceleration, v_r is the relative speed, and DX_{des} represents the desired space gap. This model emphasizes the importance of maintaining safe vehicle distances based on speed and time headway.

In addition, [20] introduced the Full Velocity Difference Model to address shortcomings in car-following models, defined as

$$a_{n+1}(t) = [V(s) - V_{\text{opt},n+1}(t)] + \lambda \Delta V,$$

where $V(s)$ is the optimal velocity and λ the sensitivity parameter. [21] investigated urban traffic using the ChSch model, which combines the BML and Nagel-Schreckenberg models, expressed as

$$\Delta x_i = V_i + \min(a_i, V_{\max}) - 1,$$

capturing vehicle displacement and acceleration dynamics. [22] used an optimal velocity model with time lag to study traffic flow regimes, formulating

$$v(t) = V(Dx_n(t)) - d^2 \sinh^2 \left(\frac{F}{s} \right) \frac{dx_n(t)}{dt}.$$

[23] generalized this to include the headway of both the target vehicle and its predecessor, with the equation

$$a_n(t) = a(V_{\text{opt}} - \dot{x}_n(t)).$$

Meanwhile, [24] highlighted the significance of backward-looking behavior in traffic modeling with the BL-OV model, expressed as

$$a(t) = lV_{\text{opt}}(V(x_{n+1} - x_n) - G \frac{dx_n}{dt}),$$

where l and G are sensitivity constants.

Further studies by [25] simulated microscopic traffic flow, providing a model for vehicle movement as

$$c(t+h) = x(t) + v(t) \cdot h + 0.5 \cdot a(t) \cdot h^2.$$

This captures the relationship between position, velocity, and acceleration. The longitudinal characteristics in road traffic to improve management and safety were explored by [26], who modeled vehicle interaction as

$$\ddot{x}_j(t+\tau) = l_j \cdot \frac{(V_i(t) - V_j(t))^k}{(X_i(t) - X_j(t))^m},$$

where l_j is the sensitivity factor of the following driver. On the other hand, [27] incorporated driver reaction time into the optimal velocity model for traffic flow, formulating

$$a(t) + v_n(t) = V(H_s(t - \tau_d)),$$

with H_s representing the headway evaluated at a delayed time. Meanwhile, [28] investigated automatic cruise control (ACC) systems, formulating the equation

$$a_n^{\text{ACC}} = \left(1 - \frac{v_n}{K_1 g_n} - \frac{v_n - 1}{K_2 g_{n-1}} + \theta^{-1} \right),$$

where g_n is the gap to the vehicle ahead, and K_1, K_2 are adaptation coefficients.

Interestingly, statistical mechanics were employed to solve traffic flow problems using cellular automata [29], while the response of drivers to cut-in events on motorways was analyzed by [30], with the model

$$PBCK = \beta_0 + \beta_1 \cdot TTC + \beta_2 \cdot DV_0 + \beta_3 \cdot DX + \beta_4 \cdot LOOM + \epsilon,$$

examining the effect of factors such as time to collision (TTC) and speed difference.

Following this, [31] analyzed the stability of vehicle platoons using experimental data with the response model

$$\text{Response}(t) = \text{stimuli}(t - T) + \tau,$$

incorporating reaction delay and sensitivity in car-following behavior. In the same year, anticipatory driving behavior was explored using gap dynamics models [32]; car-following models were refined to incorporate the effects of acceleration-deceleration asymmetry [33]; simulations of pedestrian dynamics in evacuation scenarios were analyzed, accounting for friction and its impact on evacuation times, using an iterative model

$$a(t) = a(t-1) + za(t-2).$$

[34] and [35] provided a quantitative framework for assessing the impact of different parameters on travel time and for selecting parameter sets that best represent real-world conditions.

Traffic flow dynamics, especially congestion, were modeled through headway-dependent optimal velocity functions, emphasizing the role of preceding vehicle's headway in vehicle acceleration [36]. During this same time, vehicle platoon behavior was analyzed through root-mean-square (RMS) error calculations between simulation data and field observations, contributing to the refinement of transportation simulation models [37]; [38] replicated real-world car-following behavior based on empirical data collected from experiments conducted on a test track in Japan; as [39] investigated and extended the floor field model, a cellular automaton model used for studying evacuation dynamics, focusing on improving the realism of evacuation simulations, especially in panic situations.

Between 2005 and 2006, significant advancements were made in modeling traffic flow and vehicle dynamics, particularly in car-following behavior and lane-changing dynamics. One notable contribution was the investigation of car-following dynamics using real-time kinematic (RTK) GPS data, leading to the development of an acceleration model where the acceleration of the following vehicle, $a_n(t_n + T)$, depends on the relative speed between it and the preceding vehicle, given by

$$a_n(t_n + T) = \alpha v_n(t_n + T) \left(1 - \frac{1}{m} v_r(t_n) \right).$$

In [40]. Other researchers introduced Kalman filters to estimate the speed profiles of vehicles in platoons, represented by the equation

$$\hat{x}_k = A_k \hat{x}_{k-1} + B_k u_k + K_k (y_k - C_k \hat{x}_k),$$

ensuring consistency in traffic data collected during car-following studies [41].

Further developments included stochastic cellular automaton models for traffic flow, such as the asymmetric simple exclusion process (ASEP) and zero-range process (ZRP) extensions, used to simulate traffic flow and vehicle behavior in congested conditions [42]. Route choice functions in dynamic traffic assignment simulations were also improved to enhance model accuracy, particularly through refined estimation of scale factors [43].

In 2006, traffic models incorporating vehicle gaps were developed [44], focusing on how the speed of a vehicle $\frac{dv}{dt}$ adjusts based on the gap between vehicles g and the optimal speed $V(g)$, leading to the equation:

$$\frac{dv}{dt} = A(g, v)(V(g) - v).$$

A stochastic term was introduced into the car-following behavior model, described by the equation

$$a_n(t) = \frac{d}{dt} v_n(t) = f_{min}(\Delta v_n(t)) + g(t + \tau_n),$$

where $\Delta v_n(t)$ is the relative velocity between the leading and following vehicles [45]. The Human Driver Model incorporated stochastic decision-making processes into car-following models, improving the realism of traffic simulations with the equation

$$\dot{V}_\alpha = f(s_\alpha, V_\alpha, \Delta v_\alpha, \text{other parameters}) + \text{stochastic term},$$

where $\dot{V}_\alpha$ represents the rate of change of the vehicle's velocity [46].

Besides, queuing network models also saw significant developments during this period. In models with infinite buffer sizes, the equation

$$c_{Am}^2 = \frac{1}{\lambda_m} \left\{ \sum_{t=1}^T \left[\lambda_t P_{t,m}^2 (1 - \rho_t^2) c_{At}^2 + \lambda_t P_{t,m} \left(P_{t,m} \rho_t^2 c_{St}^2 + 1 - P_{t,m} + \lambda'_m c_{Am}^2 \right) \right] \right\}$$

was introduced to explore the impact of buffer size on system performance [47].

To improve the predictive accuracy in traffic flow, methodology for short-term traffic volume forecasting was proposed using wavelet decomposition and Kalman filter models [48], described by the state prediction equation

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1},$$

while [49] proposed a methodology that derives traffic stream models from car-following models and fits these models to stationary traffic data. A satisfaction-based model for lane-changing behavior in heavy vehicles was introduced, with the satisfaction level σ given by

$$\sigma = v_{\text{old}} \left(\frac{v_{\text{new}}/\text{lim}(v) \cdot \epsilon}{(v_{\text{new}}/\text{lim}(v)) + \epsilon} \right),$$

providing insights into driver decision-making during lane changes [50].

A significant contribution by [51] introduced a systematic methodology for calibrating time-dependent Origin-Destination (O-D) matrices in microscopic traffic simulations; with [52] focusing on the impact of braking and acceleration during lane changes. The equation

$$\frac{\delta V_x}{\delta t} = a_{\text{max}} \left(1 - \left(\frac{V_x}{V_d} \right)^4 - \left(\frac{s^*}{s_\alpha} \right)^2 \right),$$

describes how the velocity of a vehicle V_x changes as a function of time, acceleration, and the gap to the preceding vehicle. In other respects, [53] proposed a framework for decision-making in traffic conditions using the acceleration model

$$a_n(t) = \frac{\alpha v_n(t)^\beta}{\Delta x_n(t - T_n)},$$

where the vehicle's acceleration $a_n(t)$ depends on the speed $v_n(t)$, spacing, and reaction time to the lead vehicle. [54] contributed to traffic performance models, modeling the relationship between traffic outflow, link capacity, and traffic density; with advances in short-term traffic forecasting [55].

Other innovations in traffic modeling during this period include the development of autonomous vehicle platoon control strategies by [56], where the acceleration equation

$$a = a_{\text{min}} \left(1 - \left(\frac{V_c}{V_d} \right)^\delta - \left(\frac{s^*}{s} \right)^2 \right),$$

was introduced to control vehicle spacing and velocity in a platoon. In a related study, [57] proposed an Adaptive Cruise Control (ACC) strategy aimed at improving traffic flow using the Intelligent Driver Model (IDM). The ACC model

$$\frac{\delta v_x}{\delta t} = a_m \left(1 - \left(\frac{v_x}{v_0} \right)^4 - \left(\frac{s^*}{h} \right)^2 \right),$$

accounts for driver behavior and desired safety gaps.

Calibration efforts in traffic flow simulations were enhanced by [58], who proposed a model for calculating the error between simulated and observed speed data, providing a robust framework for simulation accuracy; [59] implemented a framework for evaluating the microscopic properties of traffic flow; and [60] developed a model for estimating vehicle fuel consumption, considering factors such as velocity, acceleration, and road slope. This model,

$$P(V, a) = P_0 + V \cdot m(a + (\mu_0 + \mu_1 V + \beta)g) + \frac{1}{2} c_w \rho A V^2,$$

accounts for both the mechanical and aerodynamic forces acting on a vehicle.

In another study, [61] compared the Intelligent Driver Model (IDM) and the Velocity Difference Model (VDIFF), utilizing radar sensor data to calibrate the models, with acceleration defined by

$$a_{VDIFF} = v_{\text{opt}}(s) - \frac{v}{\tau} - \lambda \Delta v,$$

where τ is the relaxation time and λ the sensitivity parameter. Traffic behavior during lane changes was also studied by [62], who measured lane-changing duration with the equation

$$T_{lc} = \tau_e - \tau_s,$$

indicating the time between the start and end of lane changes.

Also, cognitive decision-making in traffic modeling was explored by [63], who introduced a car-following model based on cognitive aspects, incorporating decision-making under uncertainty and risk-taking tendencies. The model utilizes a probability function

$$f(a) = \frac{\exp(\beta \cdot U(a))}{\int_{a_{\min}}^{a_{\max}} \exp(\beta \cdot U(a')) da'}$$

to reflect driver behavior.

Interestingly, [64] developed a kinetic model to describe single-lane traffic flow, where changes in velocity and headway between vehicles are captured by the equations

$$\frac{dv_i}{dt} = f(v_{i-1}, v_i, h_i), \quad \frac{dh_i}{dt} = g(v_{i-1}, v_i, h_i);$$

[65] provided a comprehensive overview of agent-based traffic simulations, using an object-oriented approach, with vehicle acceleration modeled as

$$\frac{dv_\alpha}{dt} = f(s_\alpha, v_\alpha, \Delta v_\alpha);$$

whereas [66] developed a statistical model predicting vehicle-following headway, represented by the regression equation

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n + \epsilon.$$

Between 2010 and 2011, numerous advancements were made. For example, [67] proposed a model for evaluating weaving segments, given by

$$L_t - C_a = \beta_0 + \beta_1 \cdot L_{ws} + \beta_2 \cdot W_r + \beta_3,$$

where variables such as lane count, weaving activities, and section length determine traffic capacity. [68] analyzed the effects of fog on driving behavior, utilizing a speed-change model

$$\dot{v}_{ii}(t) = p_s \left(\frac{v_{i-1}(t) - v_{ii}(t) - h}{\Delta x_{i-1,i}(t)} \right),$$

where the spacing between vehicles and the driver's sensitivity impact traffic stability. [69] focused on the perturbation and stability of intelligent driver models (IDM) in multi-anticipatory car-following behavior, represented by

$$v_i(t) = \dot{v}_i \left(1 - \left(\frac{v_i(t)}{v_0} \right)^\delta - \left(\frac{s^*}{\Delta x_{i-1,i}(t)} \right)^2 \right);$$

with [70] exploring traffic flow stability and emergent behavior through a nonlinear model

$$\dot{V}_n = f(h_n, V_n, V_{n+1}).$$

In the realm of adaptive driving technologies, [71] proposed an enhanced Adaptive Cruise Control (ACC) model that adapts to varying traffic conditions. The ACC model incorporates the equation

$$a_{ACC} = \begin{cases} a_{IDM}, & \text{if } a_{IDM} \geq a_{CAH}, \\ (1-c)a_{IDM} + c \cdot a_{CAH} + D_{com} \cdot \tanh\left(\frac{a_{IDM} - a_{CAH}}{D_{com}}\right), & \text{if } a_{IDM} < a_{CAH}, \end{cases}$$

where a_{IDM} and a_{CAH} represent accelerations based on different driving models. Furthermore, the authors evaluated the impact of Adaptive Cruise Control (ACC) systems in mixed-traffic environments using a control model

$$a = a_{\min} \left(1 - \left(\frac{V_c}{V_d} \right)^\delta - \left(\frac{s^*}{s} \right)^2 \right),$$

focusing on vehicle interactions and safety.

The Optimal Velocity (OV) model was extended by introducing the honk effect, which modifies driver behavior when hindered by the vehicle ahead [72]. This effect is described by the equation

$$\frac{dV_n(t)}{dt} = V_{\text{opt}} - v_n \cdot \frac{\tau}{\tau'} + \lambda \cdot \mu_n \cdot \frac{v_{\text{max}} - v_n}{\tau'},$$

where λ and μ_n represent the honk effect coefficients.

It is also worth noting [73] microscopic traffic simulator for urban and highway environments. The probability of selecting a route was modeled as

$$p(R_j) = \frac{U_{kj}}{U_{ki}},$$

where U_{kj} and U_{ki} are the utilities of different routes. This highlights the decision-making process of drivers in selecting the optimal path.

Moving to 2011, [74] expanded on the idea of open-system traffic dynamics by introducing noise and bottlenecks with a stochastic model

$$a_\alpha = a_{\text{mic}}[s_\alpha(t), V_\alpha(t), V_{\alpha-1}(t)] + \epsilon_\alpha(t) \cdot V_\alpha \delta[x_\alpha(t)],$$

incorporating random fluctuations in vehicle speeds. [75] contributed to vehicle trajectory modeling in mixed traffic, using a quadratic position equation

$$x(t) = a_0 + a_1 t + a_2 t^2,$$

to reconstruct paths. [76] presented a cognitive approach to crowd behavior, simulating pedestrian flows and crowd disasters with equations such as

$$X(\alpha) = d_{\text{max}}^2 + f(\alpha)^2 - 2d_{\text{max}}f(\alpha) \cdot \cos(\alpha_0 - \alpha),$$

to account for individual decision-making processes in dense environments.

Research in traffic dynamics continued to evolve through advanced modeling techniques. For example, [77] introduced a car-following model for controlling traffic congestion, with the acceleration of a vehicle given by

$$a_n(t) = k[V_{\text{opt}}(\Delta x_n(t)) - V_n(t)] + \lambda \cdot \Delta V_n(t).$$

In this model, $V_{\text{opt}}(\Delta x_n(t))$ is the optimal velocity function, representing how drivers adjust based on headway. Additionally, [78] formulated a novel car-following model focusing on stability, represented by

$$x_n(t+1) = \minmax([Muw(x(t))]_n + cuw_n),$$

where Muw is a transition probability matrix. [79] reviewed road network equilibrium approaches, with a model for route travel time given by

$$T_k(x_k) = \min_{j \in R_{o \rightarrow d}} T_j(x_j),$$

where $T_k(x_k)$ represents the travel time on a route, and $R_{o \rightarrow d}$ is the set of all routes between an origin and a destination.

Further advances were made by [80], who developed a framework for modeling driver interactions and safe driving behaviors, utilizing the equation

$$\frac{dv}{dt} = a \left(1 - \left(\frac{v}{v_0} \right)^\delta - \left(\frac{s^*}{s} \right)^2 \right),$$

to describe the change in vehicle velocity based on headway and desired velocity. [81] focused on city logistics models, using optimization models such as

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij},$$

to minimize travel costs. [82] provided a mathematical representation of vehicle-following behavior, expressed as

$$\dot{V}_i = f(V_i(t), X_i(t), V_{i-1}(t), X_{i-1}(t)),$$

where $V_i(t)$ is the current velocity and $X_i(t)$ the current spacing between vehicles.

In urban traffic control, [83] explored computational intelligence paradigms for optimizing traffic signal control, represented by

$$\rho_k(t+1) = Q_k(t) + \sum \min(c_{kj}, R_{kj}) - \sum \min(C_{jk}, R_{jk}),$$

where $\rho_k(t)$ is the vehicle density.

From 2013 to 2014, researchers continued to enhance models addressing various aspects of traffic dynamics and vehicle interactions. For example, [84] assessed the strengths and weaknesses of existing models, providing insights for future improvements. [85] investigated U-turn movements at T-shaped intersections, proposing a series of equations to capture vehicle movements such as

$$V(t+1) = \min(v_n(t) + 1, V_{\max}),$$

where $V_{\max}$ represents the maximum allowed velocity. [86] addressed air pollution from road transport, modeling emissions using

$$E_i = \sum (V_j \cdot D_j) \cdot E_{ij,km},$$

where E_i is the emission of a specific compound, V_j is the number of vehicles, and D_j is the distance traveled.

In traffic flow and car-following behavior, [87] provided a framework to predict traffic flow dynamics, capturing individual vehicle acceleration. [88] introduced a novel continuous OV car-following model, analyzing its stability and dynamics, and exploring its behavior under different optimal speed functions.

In the realm of vehicular platoons, [89] introduced a string stability model using Connected Cruise Control (CCC), represented by the equation

$$\dot{X}(t) = AX(t) + BX(t) + CX(t-s) + DX(t-s) + E\dot{X}(t-r_1),$$

where $X(t)$ is the state vector for the platoon.

In the realm of connected and automated vehicles, [90] enhanced the safety of Cooperative Adaptive Cruise Control (CACC) systems by incorporating security measures, modeled as

$$a_i(t) = k_V(V_{i-1}(t) - V_i(t)) + k_X(X_{i-1}(t) - X_i(t) - X_0).$$

In the same year, [91] assessed microscopic traffic models, especially their ability to predict fuel consumption and emissions. [92] explored maritime traffic complexity through a model incorporating traffic density, given by

$$\text{den}_{ij} = \rho(D_{ij}) = \lambda e^{\alpha R_{ij}},$$

where R_{ij} represents the minimum safe distance between vessels. [93] developed a platoon management protocol for Connected and Automated Vehicles (CAVs), defining throughput as

$$Q = \frac{V \cdot N}{V \cdot T_g(N-1) + V \cdot T_p + N \cdot (L_v + G_{\min})},$$

where V is velocity, N is the number of vehicles, and L_v is vehicle length.

[94] proposed a model that challenges traditional car-following assumptions by incorporating both velocity and spacing sensitivities, expressed as

$$a_n(t) = \lambda(V_{n-1}(t) - V_n(t)) + \beta(x_{n-1}(t) - x_n(t) - D(V_n)).$$

Meanwhile, [95] introduced a mixed-flow model for the Aimsun simulator, refining how follower vehicles adjust their speed based on the leader's behavior, using

$$v_f(t) = v_f(t-1) - \min \left(\frac{v_f(t-1) - v_l(t-1)}{t_\tau}, \frac{a_{max}(t_\tau + t_{lat} - t_f)^2}{2} \right).$$

Besides, [96] sought to improve traffic flow simulations through the Intelligent Driver Model (IDM). These authors [96] presented an acceleration equation

$$\dot{v}_i = a_{max} \left(1 - \left(\frac{v_i}{v_{des}} \right)^2 - \frac{s^*(v_i, \Delta v_i)}{s_i} \right),$$

to account for safe following distances and speed differences.

Following this, researchers such as [97] developed a real-time traffic flow estimation model from aerial videos,

$$V_a = \frac{l_a}{l_p} \cdot \frac{d_p}{f},$$

enhancing observational accuracy. [98] predicted lane-changing duration through a regression-based model

$$\ln(LCD) = \beta(X) + \epsilon;$$

while [99] offered insights into follower vehicle dynamics, proposing an acceleration model

$$a_k = f(s_k, v_k, v_{Lk}, \beta).$$

Again, [100] underscored the importance of incorporating human factors such as reaction time and driving behavior into traffic simulation models, improving the predictive power of simulations through the model

$$a_i(t) = a_0 \left(1 - \left(\frac{V_i(t)}{v_0} \right)^\delta - \left(\frac{s_i(t)}{s_0} \right)^2 \right) + \Delta a_i,$$

which captures human responses to traffic conditions. In a diverse opinion, [101] proposed modifications to existing car-following models, incorporating additional sensitivity to changes in speed and distance, expressed as

$$a(t) = V(S(t)) - V(v(t)) - \frac{dV(S(t))}{dt} \cdot \frac{dS(t)}{dt} + \frac{dV(v(t))}{dt} k\lambda\dot{v}.$$

[102] addressed stochastic models for departure headway in traffic at signalized intersections, using a log-normal distribution; and [103] developed an optimization-based methodology for traffic state estimation, minimizing the sum of squared errors in vehicle count and jam density.

In the context of traffic flow oscillations, [104] investigated stop-and-go behavior with a minimal model, represented as

$$\dot{v} = f(s_n, v_n, v_l) + \epsilon_n(t),$$

capturing the influence of noise and instabilities in congested traffic. Moreover, [105] explored the impact of mobile phone distractions on driver reaction times, modeled by

$$\log T = b_0 + bX + r\epsilon,$$

where T is the reaction time following a Weibull distribution. [106] modeled marine traffic flow using a cellular automaton-based model, accounting for weather impacts on ship velocities through

$$V_j(t_s) = V_j(t_s) + a'_j(t_s) \cdot t_s + a''_j(t_s) \cdot t_s;$$

but [107] provided insights into traffic dynamics near critical points using a complex equation to model the interplay of space and time variables.

The advancements in traffic flow modeling from 2018 to 2019 introduced a range of innovative approaches that enhanced our understanding of traffic dynamics and vehicle control. Here, [108] developed a feedback control model for lattice hydrodynamics in traffic, incorporating historical optimal velocity differences to better simulate traffic behavior. The continuity equation

$$\frac{\delta p_j}{\delta t} + p_0(q_j - q_{j-1}) = 0,$$

and the flux equation

$$\frac{\delta q_j}{\delta t} = a[p_0 V(p_{j+1}) - q_j] + A_j,$$

with feedback control signal

$$A_j = k[p_0 V(p_{j+1}) - p_0 V(p_{j+1(t-1)})],$$

showed how historical data can influence traffic control.

In the realm of pedestrian dynamics, [109] analyzed improvements to the Social Force Model, incorporating forces such as driving, repulsive, obstacle, attraction, and fluctuation, as represented by the equation

$$F_{ij} = F_{\text{drive}} + F_{\text{repulsive}} + F_{\text{obstacle}} + F_{\text{attraction}} + F_{\text{fluctuation}}.$$

On the vehicular side, [110] applied convex optimization principles to compute feasible velocity profiles for vehicles, with a model described by

$$F(V) = \sum_{i=2}^{n-1} (V_{i-1} + V(i+1) - 2V_i)h^2 A_j,$$

offering a systematic approach to optimizing vehicle motion along complex paths.

Several other contributions focused on improving the accuracy and practicality of traffic flow models. In the same year, [111] utilized UAVs to estimate real-time traffic flow parameters, with the model

$$\text{Volume} = \text{speed} \times \text{density} \times \text{NOL};$$

[112] reviewed stability in car-following models, presenting a generalized model

$$a = a_{\min} \left(1 - \left(\frac{V_c}{V_d} \right)^\delta - \left(\frac{X^*}{X} \right)^2 \right),$$

which highlights the interaction between desired velocity and safe distance. [113] explored connected automated vehicles with models like

$$\dot{v}_0(t) = \text{sat} \left(\sum_{i=1}^n u_i(t - \sigma_i) \right),$$

focusing on control systems that utilize information beyond the immediate line of sight.

Moreover, [114] developed a discrete-time car-following model to simulate single-lane traffic, with updates defined by

$$X_{n+1} = X_n + V_n \Delta t, \quad V_{n+1} = V_n + a_n \Delta t;$$

[115] studied the stabilizing effects of autonomous vehicles on traffic flow, modeled by

$$\ddot{X}_j = f(h_j, \dot{h}_j, V_j),$$

highlighting how autonomous technology can smooth traffic conditions amidst human-driven vehicles.

[116] addressed computational inefficiencies in calibrating large-scale microscopic network models by integrating a novel analytical traffic model into a meta-model simulation–optimization (SO) approach. The model considered the interaction between multiple variables affecting vehicle velocity, using expressions like

$$V_n(t + \tau) = \min(v_{a,n}(t + \tau), v_{b,n}(t + \tau)),$$

to account for factors such as acceleration and speed limits.

On a different note, [117] extended the Cooperative Adaptive Cruise Control (CACC) model to analyze traffic flow stability, incorporating factors like optimal velocity differences and feedback from electronic throttle systems. Their extended car-following model, which includes multiple dynamic terms, reflected the complexity of vehicle interactions in modern traffic scenarios. Finally, [118] leveraged Connected and Autonomous Vehicle (CAV) technology to develop traffic control strategies using state equations like

$$x_{kk}(t + 1) = f(x_{kk}(t), u_{kk}(t), d_{kk}(t)),$$

highlighting the integration of environmental inputs in decision-making for CAVs.

In 2020, [119] emphasized the social impact of Connected and Autonomous Vehicles (CAVs) with a model evaluating the trade-off between societal benefits and individual costs. [120] created a detailed simulation model for individual vehicle movements in traffic. [121] applied game theory to analyze driver decision-making during lane changes, while [122] proposed a time-delayed feedback control strategy for stabilizing traffic flow. Meanwhile, [123] described vehicle spacing and velocity control on highways using a model based on spacing and congestion dynamics, ensuring that vehicles maintain safe distances while optimizing speed.

[124] analyzed driving dynamics, particularly in signalized intersections, where vehicle interactions are critical. The model described the evolution of the system's state,

$$S_{t+1} = (1 - p)S_t + pY_t,$$

capturing how the current state and external factors influence future behavior. With that said, [125] developed a microscopic traffic model to simulate the behavior of individual vehicles within a network, using the equation

$$V(t) = V_0 + a \cdot t,$$

which accounts for the velocity change over time under constant acceleration. These developments demonstrate the growing sophistication of traffic flow models, emphasizing the integration of multiple factors, such as driver behavior, vehicle interaction, and environmental influences.

Building on these efforts, [126] presented the Bexelius car-following model, which provides insights into vehicle behavior on highways, focusing on driver reaction delays and sensitivity to speed differences, as captured by the equation

$$\dot{V} = \frac{1}{T_\tau}(V_\tau - V_l) + c(V_\tau - V_l)^2.$$

[127] addressed parameter identifiability in car-following models with equations that describe the space gap and velocity between leading and following vehicles:

$$\dot{X}(t) = V(t) - V_l(t), \quad \dot{V}(t) = u(t) - V(t), \quad y(t) = [X(t), V(t)]^T,$$

where $X(t)$ is the space gap, $V(t)$ is the velocity of the following vehicle, and $u(t)$ is the control input representing acceleration or deceleration.

[128] proposed a framework integrating automated vehicle technologies into traffic flow optimization, using a model defined by

$$a(t) = f_{IDM}(s(t), v(t), \Delta v(t)),$$

where $a(t)$ represents acceleration, $s(t)$ is the lead vehicle distance, $v(t)$ is vehicle speed, and $\Delta v(t)$ is the inter-vehicle speed difference.

[129] extended the modeling of car-following behavior to account for vehicle emissions, presenting a more realistic approach to modeling traffic flow in complex urban environments. The model estimates speed with the equation

$$V_n(t + \tau_{n,t}) = \min \left(V_{\max}, V_n(t) + a_n(t)\tau_{n,t}, \frac{\Delta x_n(t)}{\delta_{n,t}\tau_{n,t}} \right),$$

where $V_n(t)$ is the speed of the following vehicle, $\tau_{n,t}$ is its response time, $V_{\max}$ is the maximum speed, $a_n(t)$ is the acceleration, $\Delta x_n(t)$ is the vehicle spacing, and $\delta_{n,t}$ represents the critical jam spacing.

The simulation of autonomous and connected vehicles also saw some expansions [130,131]. [130] introduced a model using

$$\frac{dv}{dt} = a_{\max} \left(1 - \left(\frac{v}{v_d} \right)^\delta \right) - b(X^* - X) - c(v - v_{ahead}),$$

where v is the vehicle speed, $a_{\max}$ is the maximum acceleration, v_d is the desired speed, δ is a shaping coefficient, and X^* and X represent the desired and actual headway, respectively.

In recent years, research on traffic models continues to evolve with a focus on understanding driving behavior, vehicle interactions, and optimizing traffic flow. In the area of traffic loads, [132] explored neural network models for recognizing traffic loads, with an example forward pass equation

$$z^l = W^l a^{l-1} + b^l,$$

where W^l is the weight matrix and b^l is the bias vector. The backward pass uses gradient descent for optimization.

In another context, [133] contributed to the field by offering models such as

$$\frac{dv}{dt} = a \left(1 - \left(\frac{v}{v_0} \right)^\delta - \left(\frac{s^*}{s} \right)^2 \right),$$

which simulate vehicle behavior for traffic safety and efficiency. [134] developed a traffic model addressing vehicle interactions and lane-changing behavior through the system of equations

$$\dot{X}_i = V_i, \quad \dot{V}_i = \alpha(V(\Delta X_i) - v_i) + \beta \frac{V_{i+1} - V_i}{(X_{i+1} - X_i - l_V)^2},$$

where α and β are model parameters, X_i and V_i represent the position and velocity of the i -th vehicle, and $V(\Delta X_i)$ is the optimal velocity function. Additionally, [135] introduced a decentralized traffic management system leveraging reinforcement learning (RL) to improve traffic efficiency. The model used in their study is

$$a = a_{\max} \left(1 - \left(\frac{V}{V_{free}} \right)^\delta - \left(\frac{s_0 + vH_s}{s - x - l} \right)^2 \right),$$

where a represents vehicle acceleration, V is vehicle speed, and other parameters such as $a_{\max}$, V_{free} , and H_s control the system's response to traffic conditions.

Nonetheless, [136] addressed the challenge of estimating missing traffic volume on urban road networks by employing a multi-source data fusion approach, while [137] discussed the challenges and opportunities for traffic flow modeling in the era of CAVs and human-driven vehicles.

Enhanced models for safety-critical situations and stochastic behavior in traffic were explored by [138, 139]. [138] introduced an optimal velocity model

$$V_{opt}(s) = v_d \tanh\left(\frac{s}{s - \beta}\right) + \tanh(\beta),$$

while [139] proposed a stochastic extension of Newell's car-following model

$$v_n(t + \Delta t) = \bar{v}_n(t + \Delta t) + \epsilon_n(t + \Delta t),$$

where $\epsilon_n(t + \Delta t)$ adds a stochastic term to the vehicle's revised speed $\bar{v}_n(t + \Delta t)$.

On a different note, [140] investigated multi-agent systems with applications to traffic flow, using a model where the agent velocity is updated according to

$$\dot{x}(t) = v_i(t), \quad v_i(t) = \frac{1}{N} \sum_{j=1}^N a \|x_j(t) - x_i(t)\| (v_j(t) - v_i(t)),$$

representing the dynamics of vehicles' positions and velocities. The author [141] again addressed the inefficiencies of static traffic light control systems by proposing a dynamic system based on reinforcement learning, using the model

$$\dot{v}_i = a \left[1 - \left(\frac{v_i}{v_{des,i}} \right)^\delta - \left(\frac{s_i^*(v_i, \Delta v_{i,j})}{s_i} \right)^2 \right],$$

where δ is the acceleration exponent, $s_i^*(v_i, \Delta v_{i,j})$ is the desired gap to the leading vehicle, and s_i is the actual gap.

These models provide a foundation for optimizing traffic flow and improving the efficiency of modern transportation systems.

3. Trends

The trends in microscopic traffic models from the 1950s to 2023 illustrate significant advancements and paradigm shifts within the field. The foundational period of the 1950s and 1960s laid the groundwork for vehicle-following theories, marked by seminal contributions such as Pipes' operational model in 1953. This model established the core principles for understanding safe following distances among vehicles. The early explorations, including Prigogine's Boltzmann-like model, drew from statistical mechanics to provide a theoretical framework for traffic dynamics [6]. However, the limited availability of empirical data during this period restricted the scope of research, resulting in a low publication output.

The 1980s and 1990s witnessed a gradual expansion of the field as researchers began to incorporate behavioral aspects into traffic models. Gipps' 1980 car-following model introduced behavioral rules that simulated vehicle interactions, signifying a shift towards more realistic traffic simulations. The diversification of research during this era reflects an interdisciplinary approach to traffic modeling. This period marked an increase in published works, signaling a growing interest in simulation techniques and their applications in traffic management.

Entering the 2000s, the field experienced exponential growth, driven by advancements in computational power and the emergence of new technologies. Key studies, such as those by [16], showcased the development of advanced car-following models. This decade was characterized by the rise of Advanced Driver Assistance Systems (ADAS), which provided real-world applications for theoretical models, enabling researchers to address complex issues such as traffic congestion and pedestrian safety. Moreover, the adoption of machine learning techniques began to influence traffic flow predictions, setting the stage for more automated traffic management solutions.

The 2010s marked a peak in research activity, with a significant focus on Connected Autonomous Vehicles (CAVs) and adaptive driving technologies. Notable studies, such as those investigating adaptive cruise control systems [71], played a crucial role in transitioning from traditional traffic models to systems that leverage real-time data for dynamic decision-making. Additionally, sustainability emerged as a pivotal theme, with

researchers increasingly integrating environmental factors such as emissions and fuel consumption into their models, reflecting the broader societal push towards sustainable transportation solutions.

Research in the 2020s continues to evolve, building upon the momentum of previous decades with an emphasis on smart city technologies and the integration of CAVs into urban traffic systems. Recent studies, such as those by [125,137], tackle contemporary challenges in real-time traffic prediction and lane management within multi-agent systems. The integration of artificial intelligence and real-time data is increasingly vital, indicating a future where traffic models are not only predictive but also prescriptive, offering dynamic solutions to the multifaceted challenges of modern urban traffic.

4. Future Directions

The future of microscopic traffic modeling research is poised to be significantly influenced by advancements in technology, the challenges of urbanization, and the urgent need for sustainable transportation solutions. One of the most promising areas for exploration is the integration of artificial intelligence (AI) and machine learning (ML) into traffic models. These technologies will facilitate the development of sophisticated traffic prediction, optimization, and management systems. As vast amounts of real-time data from sensors, cameras, and connected devices become increasingly available, AI-driven traffic models will learn from and adapt to changing conditions, enhancing the accuracy and responsiveness of traffic management systems. This will contribute to the creation of dynamic and efficient urban mobility systems that can better accommodate the complexities of modern traffic.

Another critical direction for future research is the incorporation of connected and autonomous vehicles (CAVs) into traffic models. As the prevalence of autonomous vehicles rises, traffic systems will need to account for mixed environments where human-driven and autonomous vehicles coexist. The integration of vehicle-to-vehicle (V2V) communication and autonomous driving technologies will present new challenges and opportunities for optimizing traffic flow, enhancing safety, and mitigating congestion. Future models should focus on facilitating interactions among different types of vehicles, ensuring smooth traffic flow, and minimizing disruptions in mixed traffic scenarios.

Microscopic traffic models will also increasingly align with smart city frameworks, characterized by the interconnection of various urban systems, including transportation, energy, and public services. In smart cities, traffic models will operate in conjunction with intelligent infrastructure, such as adaptive traffic signals and real-time monitoring systems, to optimize urban mobility. The objective will be to alleviate congestion, shorten travel times, and establish more efficient transportation networks that respond dynamically to real-time conditions. Moreover, integrating traffic models into smart city systems will contribute to reducing the environmental impact of urban transport.

Environmental sustainability will occupy an increasingly central role in future traffic modeling research. As global efforts to curb carbon emissions intensify, traffic models must address the environmental repercussions of various transportation strategies. This will involve optimizing traffic flows to minimize fuel consumption and greenhouse gas emissions while incorporating alternative energy vehicles, such as electric and hydrogen-powered cars, into traffic simulations. Future models should consider not only traffic efficiency but also the broader environmental implications of traffic management decisions.

The exploration of multi-agent systems and crowd dynamics will remain a vital research area. As urban populations expand, the complexity of interactions among diverse road users, including vehicles, pedestrians, and cyclists, will increase. Future models could delve deeper into these interactions, seeking to understand how agents behave in crowded or congested environments. Research on pedestrian behavior during emergencies, public gatherings, and high-traffic scenarios will enhance safety protocols and evacuation strategies. In vehicular contexts, multi-agent systems will simulate complex interactions to improve the overall reliability and efficiency of traffic systems.

Real-time, data-driven models will be a pivotal focus for future research. The growing availability of real-time data from GPS, vehicle sensors, and connected devices opens new avenues for developing models capable of making real-time decisions. Such data-driven models will enable more accurate predictions of traffic flow, congestion, and delays, allowing traffic management systems to respond swiftly to emerging issues. This approach will be particularly crucial in dynamic urban environments, where traffic patterns can fluctuate rapidly.

Finally, as advancements in autonomous vehicle technology continue to evolve, we can anticipate the emergence of fully autonomous traffic management systems. These systems will operate with minimal human intervention, leveraging AI and machine learning algorithms to autonomously manage traffic flow. By predicting and addressing potential traffic issues proactively, autonomous traffic systems will optimize vehicle flow, alleviate congestion, and enhance overall traffic efficiency.

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