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On dominator and total dominator coloring of duplication corresponding corona of path, pan, complete and sunlet graphs

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Abstract: A dominator coloring of a graph $\mathscr{G}$ is a proper coloring where each vertex of $\mathscr{G}$ is within the closed neighborhood of at least one vertex from each color class. The minimum number of color classes required for a dominator coloring of $\mathscr{G}$ is termed the dominator chromatic number. Additionally, a total dominator coloring of a graph $\mathscr{G}$ is a proper coloring in which every vertex dominates at least one color class other than its own. The minimum number of color classes needed for a total dominator coloring of $\mathscr{G}$ is known as the total dominator chromatic number. In this paper, our objective is to derive findings concerning dominator and total dominator coloring of the duplication corresponding corona of specific graphs.

Keywords: dominator chromatic number, total dominator chromatic number, duplication corresponding corona, pan, complete graph, sun let graph

MSC: 05C15.

1. Introduction

A Graph theory has indeed emerged as a foundational discipline with far-reaching implications across various fields. Its applications span diverse domains, including biochemistry, nanotechnology, electrical engineering, computer science, and operations research. The ability to model intricate systems and relationships through graphs provides invaluable insights and solutions to complex problems. The genesis of graph theory can be traced back to Euler's seminal work on the Konigsberg bridge problem in 1735. Euler's formulation of the Eulerian graph laid the groundwork for further exploration and development within the field [1]. Subsequent contributions, such as those by A.F. Mobius in 1840, introduced fundamental concepts like complete graphs and bipartite graphs, further enriching the theoretical framework of graph theory [1]. Kuratowski's theorem, proved through recreational problems, demonstrated the planarity of certain graph structures, expanding our understanding of graph properties and paving the way for deeper investigations into graph theory. These historical milestones underscore the evolution of graph theory from its inception to its current significance. By leveraging powerful combinatorial methods, graph theory continues to shape both applied and theoretical aspects of mathematics, serving as a cornerstone for problem solving and theorem proving in various disciplines.

Graphs serve as a powerful tool for modeling relationships between objects in a wide array of disciplines. To expand a bit on proper coloring, it is a concept deeply rooted in graph theory and has applications in various fields, including scheduling, map coloring, and resource allocation. In proper coloring, each vertex of the graph is assigned a color such that no two adjacent vertices share the same color. This concept is often applied to solve problems where certain constraints must be met while assigning resources or scheduling tasks. The notation provided is $f : V(\mathscr{G}) \to C$, which represents a function f that maps the vertices of a graph $\mathscr{G}$ to a collection of color classes C. References like [2–5] likely delve deeper into specific applications, algorithms, or extensions of proper coloring within graph theory, showcasing its relevance and versatility in solving discrete problems.

The concept of dominator coloring introduces additional constraints beyond traditional proper coloring to capture more intricate relationships within the graph. In a dominator coloring of a graph $\mathscr{G}$, each vertex must be in the closed neighborhood of at least one vertex of every color class. This requirement adds a layer of connectivity to the coloring scheme, ensuring that each vertex has a certain influence on the entire graph. The minimum number of color classes required to achieve dominator coloring is termed the dominator chromatic number. A total dominator coloring takes this concept further by requiring that each vertex dominate at least one color class other than its own. This condition enhances the connectivity aspect of the coloring scheme, ensuring a more distributed dominance throughout the graph. The minimum number of color classes required for a total dominator coloring is termed the total dominator chromatic number. References such as [6–13] likely delve into the theoretical properties, algorithms, and applications of dominator coloring and its variants, showcasing their significance in graph theory and related fields. The authors Michael Cary [14] et al. give the dominator coloring concepts that are studied. The authors Xiaojing Wang et al. [15] give the degree-based topological indices of the product graphs.

Our approach is also motivated by recent developments in graph resolvability and metric-based parameters. Notably, the work on mixed metric dimension and exchange properties in hexagonal nano-networks provides a foundational framework for characterizing complex structures through minimal identifiers [16]. Similarly, fault-tolerant resolvability concepts [17] and newly introduced parameters like the local edge partition dimension [18] and the mixed partition dimension [19] highlight the importance of nuanced graph invariants. Our analysis also draws from studies on graph labeling [20] and molecular structures such as the V-Phenylenic nanotube [21], bridging the gap between theoretical constructs and practical applications. Dominator and total dominator coloring can be discussed on these structures.

Let $\mathscr{G}_1$ and $\mathscr{G}_2$ be the two vertex-disjoint graphs with n_1 and n_2 vertices, respectively. Let $V(\mathscr{G}_1) \cup \mathrm{U}\ (\mathscr{G}_1)$ where $V(\mathscr{G}_1) = \{v_1, v_2, v_3, ..., v_{n_1}\}$ and $\mathrm{U}(\mathscr{G}_1) = \{x_1, x_2, x_3, , ..., x_{n_1}\}$ be the vertex set of $\mathscr{D}\mathscr{G}_1$, the duplication graph of $\mathscr{G}_1$.The vertex x_i is a duplication of v_i for each i = 1,2,..., n_1. The vertex x_i is a duplication of v_i for each $i = 1, 2, ..., n_1$. Duplication corresponding corona [22–25] of $\mathscr{G}_1$ and $\mathscr{G}_2$, denoted by $\mathscr{G}_1 \underline{\circledast} \mathscr{G}_2$, is the graph obtained from $\mathscr{D}\mathscr{G}_1$ and n_1 copies of $\mathscr{G}_2$ by making x_i and v_i adjacent to every vertex in the i - th copy of $\mathscr{G}_2$ for $i = 1, 2, ..., n_1$.References such as [22,24,25] likely explore the properties, applications, and further details of the duplication corresponding corona graph, shedding light on its significance within graph theory and related areas. The author, Renny [23], gives some results on the spectra of a new duplication-based corona of the graph and gives a clear explanation of the definition of duplication for the corresponding corona graph. The authors Renny et al. [23,24] give a new product related to the area studied. Motivated by these works, a new corona-type graph, namely, the duplication of the corresponding corona, and its adjacency, Laplacian, and signless Laplacian spectra.

In this paper, we undertake the computation of the dominator and total dominator coloring of the duplication corresponding corona of various graphs, such as path with pan, complete, and sunlet graphs. The Pan graph denotes a graph composed of vertices representing a singleton graph K_1 with a vertex connected by a bridge to a cycle C_m of vertices denoted by $p_j, 1 \leq j \leq m$. The edges of the pan graph ρ_m are defined as $E(\rho_m) = q \cup q_j : 1 \leq j \leq m$, where 'q' represents the edge that connects the singleton graph K_1 to the cycle C_m. The Sun let graph on $2n$ vertices is formed by attaching n pendant edges to the vertices of the cycle C_n. It is denoted as S_n [13]. A complete graph is characterized by having every pair of vertices connected by an edge. A complete graph with n vertices is denoted by K_n.

2. Preliminaries

This section focuses on presenting some of the preliminary results and bounds necessary for our study.

Proposition 1. [2,7] *For the path $P_n, n \geq 2$, we have*

$$\chi_d(P_n) = \begin{cases} \lceil \frac{n}{3} \rceil + 1, & when \;\; n = 2, 3, 4, 5, 7, \\ \lceil \frac{n}{3} \rceil + 2, & otherwise. \end{cases}$$

Proposition 2. [6] *Let P_n be a path of order $n \geq 2$. Then*

$$\chi_d^t(P_n) = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil - 1, & if\ n \equiv 1 (mod\ 3), \\ 2\left\lceil \frac{n}{3} \right\rceil, & otherwise. \end{cases}$$

Lemma 1. [7] *Let $\mathscr{G}$ be a connected graph. Then*

$$max\ \{\chi(G), \gamma(G)\} \leq \chi_d(G) \leq \chi(G) + \gamma.$$

Lemma 2. [6] *For any connected graph G of order n with $\delta(G) \geq 1$. Then*

$$max\ \{\chi(G), \gamma_t(G), 2\} \leq \chi_d^t(G) \leq n.$$

3. Main results

Theorem 1. *Let $n \geq 2, m \geq 3$, the dominator chromatic number of duplication corresponding corona of path P_n with pan ρ_m is*

$$\chi_d(P_n \circledast \rho_m) = \begin{cases} n+4 & if\ m\ is\ odd, \\ n+3 & if\ m\ is\ even. \end{cases}$$

Proof. Let $V(P_n) = \{p_i : 1 \leq i \leq n\}$ represent the vertices of a path graph with n vertices, and let DP_n denote the duplication of the graph P_n, where the duplicated vertices are labeled as x_i.

Consider n copies of the Pan graph ρ_m. Let $V(\rho_m) = \{a, q_j : 1 \leq j \leq m\}$ denote the vertices of the Pan graph, where a represents the vertex of the singleton graph K_1 connected to the cycle with a bridge, and q_j represents the vertices of the cycle as defined. Let $'s'$ denote the respective copy of pan graphs taken, where $1 \leq s \leq n$. The vertex set of the resultant graph $P_n \circledast \rho_m$ is defined as follows:

$$V(P_n \circledast \rho_m) = \{p_i, x_i, q_{ij}, a_i : 1 \leq i \leq n, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of a resultant graph is given by $\mid V(P_n \circledast \rho_m) \mid = n(m+3)$. The maximum and minimum degree of a graph is given by $\Delta\ (P_n \circledast \rho_m) = m+2$ and $\delta\ (P_n \circledast \rho_m) = 3$, respectively. We associate a coloring transformation with

$$\zeta : V(P_n \circledast \rho_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d(P_n \circledast \rho_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq j \leq m$.

Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{ij} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{ij} and a_i in every s^{th} copy.
- Assign the color ζ_4 to the vertices q_{ij} in every s^{th} copy when $j = m$
- Assign the color ζ_{4+s} to the vertices x_i, for $1 \leq s \leq n$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{ij} and a_i in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{ij} in every s^{th} copy.
- Assign the color ζ_{3+s} to the vertices x_i, for $1 \leq s \leq n$.

Allocation of distinct colors to the vertices x_i allows the other vertices p_i, a_i and q_{ij} in every s-th copy to dominate the vertices x_i. These vertices x_i are self-dominating.

Using more than the colors mentioned if $\chi_d(P_n \circledast \rho_m) \geq n+4$ when m is odd and if $\chi_d(P_n \circledast \rho_m) \geq n+3$ when m is even we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d(P_n \circledast \rho_m) \leq n+4$ when m is odd and if, $\chi_d(P_n \circledast \rho_m) \leq n+3$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $n+4$ colors when m is odd and $n+3$ colors when m is even. Thus,

$$\chi_d(P_n \underline{\divideontimes} \rho_m) = \begin{cases} n+4 & if\ m\ is\ odd, \\ n+3 & if\ m\ is\ even. \end{cases}$$

□

Theorem 2. *Let $n \geq 2, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of path P_n with pan ρ_m is*

$$\chi_d^t(P_n \underline{\divideontimes} \rho_m) = \begin{cases} 2n+3 & if\ m\ is\ odd, \\ 2n+2 & if\ m\ is\ even. \end{cases}$$

Proof. We take into account all the considerations and notation introduced in the preceding theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(P_n \underline{\divideontimes} \rho_m) \to \{\zeta_1, \zeta_2, ...\chi_d^t(P_n \underline{\divideontimes} \rho_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq j \leq m$.

Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{ij} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{ij} and a_i in every s^{th} copy.
- Assign the color ζ_{3+s} to the vertices q_{ij} in every s^{th} copy when j = m.
- Assign the colors $\{\zeta_{n+4}, \zeta_{n+5}, ...\zeta_{2n+3}\}$ to the vertices x_i.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{ij} and a_i in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_{2+s} to the vertices q_{ij} in every s^{th} copy.
- Assign the color $\{\zeta_{n+3}, \zeta_{n+4}, ...\zeta_{2n+2}\}$ to the vertices x_i.

Allocation of distinct colors to the vertices x_i and q_j in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(P_n \underline{\divideontimes} \rho_m) \geq 2n+3$ when m is odd and if, $\chi_d^t(P_n \underline{\divideontimes} \rho_m) \geq 2n+2$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d^t(P_n \underline{\divideontimes} \rho_m) \leq 2n+3$ when m is odd and if, $\chi_d^t(P_n \underline{\divideontimes} \rho_m) \leq 2n+2$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $2n+3$ colors when m is odd and $2n+2$ colors when m is even. Thus,

$$\chi_d^t(P_n \underline{\divideontimes} \rho_m) = \begin{cases} 2n+3 & if\ m\ is\ odd, \\ 2n+2 & if\ m\ is\ even. \end{cases}$$

□

Theorem 3. *Let $n \geq 3, m \geq 2$, the dominator chromatic number of duplication corresponding corona of pan ρ_n with path P_m is*

$$\chi_d(\rho_n \underline{\divideontimes} P_m) = n+4.$$

Proof. Let $V(\rho_n) = \{a, p_i : 1 \leq i \leq n\}$ represent the vertices of a pan graph with $n+1$ vertices, where a represents the vertex of the singleton graph K_1 connected to the cycle with a bridge and p_i represent the vertices of the cycle as defined. Let $D\rho_n$ denote the duplication of the graph ρ_n, where the duplicated vertices are labeled as $x_t, 1 \leq t \leq n+1$.

Consider $n+1$ copies of the Path graph P_m. Let $V(P_m) = \{q_j : 1 \leq j \leq m\}$ denote the vertices of the Path graph. Let $'s'$ denote the respective copy of Path graphs taken, where $1 \leq s \leq n+1$. The vertex set of the resultant graph $\rho_n \divideontimes P_m$ is defined as follows:

$$V(\rho_n \divideontimes P_m) = \{p_i, a, x_t, q_{tj} : 1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of a resultant graph is given by $\mid V(\rho_m \divideontimes P_n) \mid = (n+1)(m+2)$. The maximum and minimum degree of a graph is given by $\Delta\ (\rho_n \divideontimes P_m) = m+3$ and $\delta(\rho_n \divideontimes P_m) = 3$ respectively. We associate a coloring transformation with

$$\zeta : V(\rho_n \divideontimes P_m) \to \{\zeta_1, \zeta_2, ...\chi_d(\rho_n \divideontimes P_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i and a.
- When $j \equiv 1 (mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} in every s^{th} copy.
- When $j \equiv 0 (mod\ 2)$, assign the color ζ_3 to the vertices q_{tj} in every s^{th} copy.
- Assign the color ζ_{3+s} to the vertices x_t, for $1 \leq s \leq n+1$.

Allocation of distinct colors to the vertices x_t allows the other vertices p_i, a and q_{tj} in every s-th copy to dominate the vertices x_t. These vertices x_t are self-dominating.

Using more than the colors mentioned if $\chi_d(\rho_n \divideontimes P_m) \geq n+4$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if $\chi_d(\rho_n \divideontimes P_m \leq n+4$, would fail to satisfy the dominator coloring property. Hence, we require $n+4$ colors. Thus,

$$\chi_d(\rho_n \divideontimes P_m) = n+4.$$

□

Theorem 4. *Let $n \geq 3, m \geq 2$, the total dominator chromatic number of duplication corresponding corona of pan ρ_n with path P_m is*

$$\chi_d^t(\rho_n \divideontimes P_m) = 2n+4.$$

Proof. We take into account all the considerations and notation introduced in the preceding theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(\rho_n \divideontimes P_m) \to \{\zeta_1, \zeta_2, ...\chi_d^t(\rho_n \divideontimes P_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i, a.
- When $j \equiv 1 (mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} in every s^{th} copy.
- When $j \equiv 0 (mod\ 2)$, assign the color ζ_{2+s} to the vertices q_{tj} in every s^{th} copy.
- Assign the color $\{\zeta_{n+4}, \zeta_{n+5}, , ..., \zeta_{2n+4}\}$ to the vertices x_t.

Allocation of distinct colors to the vertices x_t and q_{tj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(\rho_n \divideontimes P_m) \geq n+4$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if $\chi_d^t(\rho_n \divideontimes P_m) \leq n+4$, would fail to satisfy the dominator coloring property.

Hence, we require $n+4$ colors. Thus,

$$\chi_d^t(\rho_n \divideontimes P_m) = 2n+4.$$

□

Theorem 5. *Let $n, m \geq 3$, the dominator chromatic number of duplication corresponding corona of pan ρ_n with pan ρ_m is*

$$\chi_d(\rho_n \circledast \rho_m) = \begin{cases} n+5 & \text{if } m \text{ is odd,} \\ n+4 & \text{if } m \text{ is even.} \end{cases}$$

Proof. Let $V(\rho_n) = \{a, p_i : 1 \leq i \leq n\}$ represent the vertices of a pan graph with $n+1$ vertices, where a represents the vertex of the singleton graph K_1 connected to the cycle with a bridge and p_i represent the vertices of the cycle as defined. Let $D\rho_n$ denote the duplication of the graph ρ_n, where the duplicated vertices are labeled $x_t, 1 \leq t \leq n+1$.

Consider $n+1$ copies of the pan graph P_m. Let $V(\rho_n) = \{b, q_j : 1 \leq j \leq m\}$ denote the vertices of the Pan graph ρ_m, where b represents the vertex of the singleton graph K_1 connected to the cycle with a bridge and q_j represents the vertices of the cycle as defined. Let $'s'$ denote the respective copy of Pan graphs taken, where $1 \leq s \leq n+1$.The vertex set of the resultant graph $\rho_n \circledast P_m$ is defined as follows:

$$V(\rho_n \circledast \rho_m) = \{a, p_i, x_t, q_{tj}, b_t : 1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of a resultant graph is given by $|\ V(\rho_m \circledast \rho_m)\ | = (n+1)(m+3)$. The maximum and minimum degree of a graph is given by $\Delta\ (\rho_n \circledast \rho_m) = 3(m+1)$ and $\delta(\rho_n \circledast \rho_m) = 3$ respectively. We associate a coloring transformation with

$$\zeta : V(\rho_n \circledast \rho_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d(\rho_n \circledast \rho_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m$.

Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i and a.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{tj} and b_t in every s^{th} copy.
- For j=m, assign the color ζ_4 to the vertices q_{tj} in every s^{th} copies.
- Assign the color ζ_{4+s} to the vertices x_t, for $1 \leq s \leq n+1$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i and a.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} and b_t in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{tj} in every s^{th} copy.
- Assign the color ζ_{3+s} to the vertices x_t, for $1 \leq s \leq n+1$.

Allocation of distinct colors to the vertices x_t allows the other vertices in every s-th copy to dominate the vertices x_t. These vertices x_t are self-dominating.

Using more than the colors mentioned if $\chi_d(\rho_n \circledast \rho_m) \geq n+5$ when m is odd and if, $\chi_d(\rho_n \circledast \rho_m) \geq n+4$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d(\rho_n \circledast \rho_m) \leq n+5$ when m is odd and if, $\chi_d(\rho_n \circledast \rho_m) \leq n+4$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $n+5$ colors when m is odd and $n+4$ colors when m is even. Thus,

$$\chi_d(\rho_n \circledast \rho_m) = \begin{cases} n+5 & if\ m \text{ is odd,} \\ n+4 & if\ m \text{ is even.} \end{cases}$$

□

Theorem 6. *Let $n, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of pan ρ_n with pan ρ_m is*

$$\chi_d^t(\rho_n \circledast \rho_m) = \begin{cases} 2n+5 & \text{if } m \text{ is odd,} \\ 2n+4 & \text{if } m \text{ is even.} \end{cases}$$

Proof. We take into account all the considerations and notation introduced in the preceding theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(\rho_n \circledast \rho_m) \to \{\zeta_1, \zeta_2, ...\chi_d^t(\rho_n \circledast \rho_m\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq t \leq n+1, 1 \leq j \leq m$.
Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i and a.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{tj} in every s^{th} copy.
- For j = m, assign the color $\{\zeta_{3+s}$ to the vertices q_{tj} in every s^{th} copy.
- Assign the colors $\{\zeta_{n+5}, \zeta_{n+6}, ...\zeta_{2n+5}\}$ to the vertices x_t, for $1 \leq s \leq n+1$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i and a.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{tj} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_{2+s} to the vertices q_{tj} in every s^{th} copy.
- Assign the color $\{\zeta_{n+4}, \zeta_{n+5}, ...\zeta_{2n+4}\}$ to the vertices x_t, for $1 \leq s \leq n+1$.

Allocation of distinct colors to the vertices x_t and q_{tj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(\rho_n \circledast \rho_m) \geq 2n+5$ when m is odd and if, $\chi_d^t(\rho_n \circledast \rho_m) \geq 2n+4$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d^t(\rho_n \circledast \rho_m) \leq 2n+5$ when m is odd and if, $\chi_d^t(\rho_n \circledast \rho_m) \leq 2n+4$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $n+5$ colors when m is odd and $n+4$ colors when m is even. Thus,

$$\chi_d^t(\rho_n \circledast \rho_m) = \begin{cases} 2n+5 & if\ m\ is\ odd, \\ 2n+4 & if\ m\ is\ even. \end{cases}$$

□

Theorem 7. *Let $n \geq 2, m \geq 3$, the dominator chromatic number of duplication corresponding corona of path P_n with sun let S_m is*

$$\chi_d(P_n \circledast S_m) = \begin{cases} n+4 & \text{if m is odd,} \\ n+3 & \text{if m is even.} \end{cases}$$

Proof. Let $V(P_n) = \{p_i : 1 \leq i \leq n\}$ represent the vertices of a path graph with n vertices, and let DP_n denote the duplication of the graph P_n, where the duplicated vertices are labeled as x_i.

Consider n copies of the sun, let the graph S_m. Let $V(S_m) = \{q_{sj}, q'_{sj} : 1 \leq s \leq 2n, 1 \leq j \leq m\}$ be the vertices of sun let graph on 2n vertices, where $'s'$ denotes the respective copies of the sun let graphs taken. The vertex set of the resultant graph $P_n \circledast S_m$ is defined as follows:

$$V(P_n \circledast S_m) = \{p_i, x_i, q_{sj}, q'_{sj} : 1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of a resultant graph is given by $\mid V(P_n \circledast S_m) \mid = 2n(m+1)$. The maximum and minimum degree of a graph is given by $\Delta(P_n \circledast S_m) = m+3$ and $\delta(P_n \circledast S_m) = 3$ respectively. We associate a coloring transformation with

$$\zeta : V(P_n \circledast S_m) \to \{\zeta_1, \zeta_2, ...\chi_d(P_n \circledast S_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m$.
Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i.

- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the color ζ_4 to the vertices q_{sj}, when j=m
- Assign the color ζ_{4+s} to the vertices x_i, for $1 \leq s \leq n$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the color ζ_{3+s} to the vertices x_i, for $1 \leq s \leq n$.

Allocation of distinct colors to the vertices x_i allows the other vertices in every s-th copy to dominate the vertices x_i. These vertices are indeed self-dominating.

Using more than the colors mentioned if $\chi_d(P_n \underline{\circledast} S_m) \geq n+4$ when m is odd and if, $\chi_d(P_n \underline{\circledast} S_m) \geq n+3$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d(P_n \underline{\circledast} S_m) \leq n+4$ when m is odd and if, $\chi_d(P_n \underline{\circledast} S_m) \leq n+3$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $n+4$ colors when m is odd and $n+3$ colors when m is even. Thus,

$$\chi_d(P_n \underline{\circledast} S_m) = \begin{cases} n+4 & if\ m\ is\ odd, \\ n+3 & if\ m\ is\ even. \end{cases}$$

□

Theorem 8. *Let $n \geq 2, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of path P_n with sunlet S_m is*

$$\chi_d^t(P_n \underline{\circledast} S_m) = \begin{cases} 2n+3 & \text{if } m \text{ is odd,} \\ 2n+2 & \text{if } m \text{ is even.} \end{cases}$$

Proof. We take into account all the considerations and notation introduced in the preceding theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(P_n \underline{\circledast} S_m) \to \{\zeta_1, \zeta_2, ...\chi_d^t(P_n \underline{\circledast} S_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m$.
Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the color ζ_{4+s} to the vertices q_{sj}, when j=m
- Assign the colors $\{\zeta_{n+4}, \zeta_{n+5}, ...\zeta_{2n+3}\}$ to the vertices x_i, for $1 \leq s \leq n$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_{2+s} to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the colors $\{\zeta_{n+3}, \zeta_{n+4}, ...\zeta_{2n+2}\}$ to the vertices x_i, for $1 \leq s \leq n$.

Allocation of distinct colors to the vertices x_i and q_{sj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(P_n \underline{\circledast} S_m) \geq 2n+3$ when m is odd and if, $\chi_d^t(P_n \underline{\circledast} S_m) \geq 2n+2$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d^t(P_n \underline{\circledast} S_m) \leq 2n+3$ when m is odd and if, $\chi_d^t(P_n \underline{\circledast} S_m) \leq 2n+2$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $n+4$ colors when m is odd and $n+3$ colors when m is even. Thus,

$$\chi_d^t(P_n \underline{\ast} S_m) = \begin{cases} 2n+3 & if\ m\ is\ odd, \\ 2n+2 & if\ m\ is\ even. \end{cases}$$

□

Theorem 9. *Let $n \geq 3, m \geq 2$, the dominator chromatic number of duplication corresponding corona of Sun let S_n with path P_m is*

$$\chi_d(S_n \underline{\ast} P_m) = 2n+3.$$

Proof. Let $V(S_n) = \{p_i, p_i' : 1 \leq i \leq n\}$ represent the vertices of a sun let graph with 2n vertices, and let DS_n denote the duplication of the graph S_n, where the duplicated vertices are labeled as x_i, x_i'.

Consider $2n$ copies of the Path graph P_m. Let $V(P_m) = \{q_{sj.} 1 \leq t \leq 2n, 1 \leq j \leq m\}$ be the vertices of the path graph, where $'s'$ denotes the respective copies of Path graphs taken.

The vertex set of the resultant graph $S_n \underline{\ast} P_m$ is defined as follows:

$$V(S_n \underline{\ast} P_m) = \{p_i, p_i', x_i, x_i', q_{sj.} 1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of the resultant graph is given by $\mid V(S_n \underline{\ast} P_m) \mid = 2n(m+2)$. The maximum and minimum degree of a graph is given by $\Delta\ (S_n \underline{\ast} P_m) = m+3$ and $\delta(S_n \underline{\ast} P_m) = 3$ respectively.

We associate a coloring transformation with

$$\zeta : V(S_n \underline{\ast} P_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d(S_n \underline{\ast} P_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i, p_i'.
- When $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{sj} in every s^{th} copy.
- When $j \equiv 0(mod\ 2)$, assign the color ζ_3 to the vertices q_{sj} in every s^{th} copy.
- Assign the color ζ_{3+s} to the vertices x_i, x_i', for $1 \leq s \leq 2n$.

Allocation of distinct colors to the vertices x_i, x_i' allows the other vertices in every s-th copy to dominate the vertices x_i, x_i'. These vertices are, in turn, self-dominating.

Using more than the colors mentioned if $\chi_d(S_n \underline{\ast} P_m) \geq 2n+3$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if $\chi_d(S_n \underline{\ast} P_m) \leq 2n+3$, would fail to satisfy the dominator coloring property. Thus,

$$\chi_d(S_n \underline{\ast} P_m) = 2n+3.$$

□

Theorem 10. *Let $n \geq 3, m \geq 2$, the total dominator chromatic number of duplication corresponding corona of sun let S_n with path P_m is*

$$\chi_d^t(S_n \underline{\ast} P_m) = 4n+2.$$

Proof. We take into account all the considerations and notation introduced in the previous theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(S_n \underline{\ast} P_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d^t(S_n \underline{\ast} P_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i, p_i'.
- For $j \equiv 1(mod\ 2)$, assign the color ζ_2 to the vertices q_{sj} in s^{th} copy.
- For $j \equiv 0(mod\ 2)$, assign the color ζ_{2+s} to the vertices q_{sj} in every s^{th} copy.
- Assign the colors $\{\zeta_{n+6}, \zeta_{n+7},,...,\zeta_{4n+2}\}$ to the vertices x_i.

Allocation of distinct colors to the vertices x_t, x'_t and q_{tj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(S_n \circledast P_m) \geq 4n+2$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d^t(S_n \circledast P_m) \leq 4n+2$, would fail to satisfy the dominator coloring property. Hence, we require $4n+2$ colors. Thus,

$$\chi_d^t(S_n \circledast P_m) = 4n+2.$$

□

Theorem 11. *Let $n, m \geq 3$, the dominator chromatic number of duplication corresponding corona of sun let S_n with sun let S_m is*

$$\chi_d(S_n \circledast S_m) = \begin{cases} 2n+4 & \text{if } m \text{ is odd}, \\ 2n+3 & \text{if } m \text{ is even}. \end{cases}$$

Proof. Let $V(S_n) = \{p_i, p'_i : 1 \leq i \leq n\}$ represent the vertices of a sun let graph with 2n vertices, and let DS_n denote the duplication of the graph S_n, where the duplicated vertices are labeled as x_i, x'_i.

Consider $2n$ copies of the Sun, let the graph S_m. Let $V(S_m) = \{q_{sj}, q_{sj'}.1 \leq s \leq 2n, 1 \leq j \leq m\}$ be the vertices of sun let graph on 2m vertices, where $'s'$ denotes the respective copy of sun let graphs taken.

The vertex set of the resultant graph $S_n \circledast S_m$ is defined as follows:

$$V(S_n \circledast S_m) = \{p_i, p'_i, x_i, x'_i, q_{sj}, q'_{sj} : 1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m\}.$$

The cardinality of the vertex set of a resultant graph is given by $\mid V(S_n \circledast S_m) \mid = 4n(m+1)$. The maximum and minimum degree of a graph is given by $\Delta\ (S_n \circledast S_m) = 6m$ and $\delta(S_n \circledast S_m) = 3$ respectively.

Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i, p'_i.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the color ζ_4 to the vertices q_{sj}, when j=m
- Assign the color ζ_{4+s} to the vertices x_i, x'_i, for $1 \leq s \leq 2n$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i,p'_i.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q'_{sj}, when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q'_{sj}, when j is odd.
- Assign the color ζ_{3+s} to the vertices x_i, x'_i, for $1 \leq s \leq 2n$.

Allocation of distinct colors to the vertices x_i, x'_i allows the other vertices in every s-th copy to dominate the vertices x_i, x'_i. These vertices are indeed self-dominating.

Using more than the colors mentioned if $\chi_d(S_n \circledast S_m) \geq 2n+4$ when m is odd and if, $\chi_d(S_n \circledast S_m) \geq 2n+3$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d(S_n \circledast S_m) \leq 2n+4$ when m is odd and if, $\chi_d(S_n \circledast S_m) \leq 2n+3$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $2n+4$ colors when m is odd and $2n+3$ colors when m is even. Thus,

$$\chi_d(S_n \circledast S_m) = \begin{cases} 2n+4 & if\ m\ is\ odd, \\ 2n+3 & if\ m\ is\ even. \end{cases}$$

□

Theorem 12. *Let $n, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of sun let S_n with sun let S_m is*

$$\chi_d^t(S_n \circledast S_m) = \begin{cases} 4n+3 & \text{if } m \text{ is odd}, \\ 4n+2 & \text{if } m \text{ is even}. \end{cases}$$

Proof. We take into account all the considerations and notations introduced in the preceding theorem. Similarly, associate a coloring transformation with

$$\zeta : V(S_n \divideontimes S_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d^t(S_n \divideontimes S_m)\}$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq s \leq 2n, 1 \leq j \leq m$.

Case 1. m is odd

- Assign the color ζ_1 to the vertices p_i,p_i'.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q_{sj}', when j is even.
- Assign the color ζ_3 to the vertices q_{sj}, when j is even and q_{sj}', when j is odd.
- Assign the color ζ_{3+s} to the vertices q_{sj}, when j=m
- Assign the colors $\{\zeta_{2n+4}, \zeta_{2n+5}, ...\zeta_{4n+3}\}$ to the vertices x_i, x_i', for $1 \leq s \leq 2n$.

Case 2. m is even

- Assign the color ζ_1 to the vertices p_i,p_i'.
- Assign the color ζ_2 to the vertices q_{sj}, when j is odd and q_{sj}', when j is even.
- Assign the color ζ_{2+s} to the vertices q_{sj}, when j is even and q_{sj}', when j is odd.
- Assign the colors $\{\zeta_{2n+3}, \zeta_{2n+4}, ...\zeta_{4n+2}\}$ to the vertices x_i, x_i', for $1 \leq s \leq 2n$.

Allocation of distinct colors to the vertices x_i, x_i' and q_{sj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(S_n \divideontimes S_m) \geq 4n+3$ when m is odd and if, $\chi_d^t(S_n \divideontimes S_m) \geq 4n+2$ when m is even, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if, $\chi_d^t(S_n \divideontimes S_m) \leq 4n+3$ when m is odd and if, $\chi_d^t(S_n \divideontimes S_m) \leq 4n+2$ when m is even, would fail to satisfy the dominator coloring property.

Hence, we require $4n+3$ colors when m is odd and $4n+2$ colors when m is even. Thus,

$$\chi_d^t(S_n \divideontimes S_m) = \begin{cases} 4n+3 & if\ m\ is\ odd, \\ 4n+2 & if\ m\ is\ even. \end{cases}$$

□

Theorem 13. *Let $n \geq 2, m \geq 3$, the dominator chromatic number of duplication corresponding corona of path P_n with complete graph κ_m is*

$$\chi_d(P_n \divideontimes \kappa_m) = m+n+1.$$

Proof. Let $V(P_n) = \{p_i : 1 \leq i \leq n\}$ represent the vertices of a path graph with n vertices, and let DP_n denote the duplication of the graph P_n, where the duplicated vertices are labeled as x_i.

Consider n copies of the complete graph κ_m. Let $V(\kappa_m) = \{q_{sj} : 1 \leq s \leq n, 1 \leq j \leq m\}$ be the vertices of complete graph κ_m, where $'s'$ denotes the respective copy of Path graphs taken. The vertex set of the resultant graph $(P_n \divideontimes \kappa_m)$ is defined as follows: $V(P_n \divideontimes \kappa_m) = \{p_i, x_i, q_{sj}. 1 \leq i, s \leq n, 1 \leq j \leq m\}$

The cardinality of the vertex set of a resultant graph is given by $\mid V(P_n \divideontimes \kappa_m) \mid = n(m+2)$.

The maximum and minimum degree of a graph is given by $\Delta(P_n \divideontimes \kappa_m) = m+3$ and $\delta(P_n \divideontimes \kappa_m) = 3$, respectively. We associate a coloring transformation with

$$\zeta : V(P_n \divideontimes \kappa_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d(P_n \divideontimes \kappa_m)\}.$$

This mapping is defined for the values $1 \leq i \leq n, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i.
- Assign the color $j+1$ to the vertices q_{sj} in every s^{th} copy.
- Assign the colors $\{\zeta_{m+2}, \zeta_{m+3}, ..., \zeta_{m+n+1}\}$ to the vertices x_i.

Allocation of distinct colors to the vertices x_i allows the other vertices in every s-th copy to dominate the vertices x_i. These vertices are indeed self-dominating.

Using more than the colors mentioned if $\chi_d(P_n \circledast \kappa_m) \geq m+n+1$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if $\chi_d(P_n \circledast \kappa_m) \leq m+n+1$, would fail to satisfy the dominator coloring property. Thus,

$$\chi_d(P_n \circledast \kappa_m) = m+n+1.$$

□

Theorem 14. *Let $n \geq 2, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of P_n with κ_m is*

$$\chi_d^t(P_n \circledast \kappa_m) = 2n+m.$$

Proof. . We take into account all the considerations and notation introduced in the preceding theorem. Similarly, we associate a coloring transformation with

$$\zeta : V(P_n \circledast \kappa_m) \rightarrow \{\zeta_1, \zeta_2, ...\chi_d^t(P_n \circledast \kappa_m)\}.$$

This mapping is defined for the values $1 \leq i, s \leq n, 1 \leq j \leq m$.

- Assign the color ζ_1 to the vertices p_i.
- For $1 \leq j \leq m-1$, assign the color $j+1$ to the vertices q_{sj} in every s-th copy.
- For $j = m$, assign the color $m+s$ to the vertices q_{sj} in every s-th copy.
- Assign the color ζ_{m+n+i} to the vertices x_i.

Allocation of distinct colors to the vertices x_i and q_{sj} in every copy allows the vertices to dominate at least one color class other than their own.

Using more than the colors mentioned if $\chi_d^t(P_n \circledast \kappa_m) \geq 2n+m$, we will arrive at a maximum number of colors. Conversely, using fewer colors, say if $\chi_d^t(P_n \circledast \kappa_m) \leq 2n+m$, would fail to satisfy the dominator coloring property. Thus,

$$\chi_d^t(P_n \circledast \kappa_m) = 2n+m.$$

□

Theorem 15. *Let $n \geq 3, m \geq 2$, the dominator chromatic number of duplication corresponding corona of complete graph κ_n with path P_m is*

$$\chi_d(\kappa_n \circledast P_m) = n+3.$$

Proof. The proof is similar to Theorem 3. □

Theorem 16. *Let $n \geq 3, m \geq 2$, the total dominator chromatic number of duplication corresponding corona of complete graph κ_n with path P_m is*

$$\chi_d^t(\kappa_n \circledast P_m) = 2n+2.$$

Proof. The proof follows from Theorem 4. □

Corollary 1. *Let $n, m \geq 3$, the dominator chromatic number of duplication corresponding corona of κ_n with κ_m is*

$$\chi_d(\kappa_n \circledast \kappa_m) = n+3.$$

Corollary 2. *Let $n, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of κ_n with κ_m is*

$$\chi_d^t(\kappa_n \circledast \kappa_m) = 2n+2.$$

4. Conclusion

In this study, we compare dominator and total dominator coloring across various corona products involving path, pan, complete, and sunlet graphs. Where the dominator and total dominator colorings have greater than or equal to or less than or equal relationships, indicating different behaviors between the two in various scenarios. The results provide a detailed analysis of dominator and total dominator coloring of graphs formed by specific coronas. The work would be extended to consider duplication corresponding to the corona of some general graphs and general graphs for some other operations, like adding a vertex and adding edge corona graphs. Finally, these results have potential real-life applications, such as network design, medical resource allocation, and optimization problems.

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