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On global solutions of the nonlinear Moore-Gibson-Thompson equation

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Abstract: This work is devoted to study the global solutions of a class of nonlinear Moore-Gibson-Thompson equation. By applying the Galerkin and compact methods, we derive some sufficient conditions on the nonlinear terms, which lead to the existence and uniqueness of the global solution.

Keywords: Moore-Gibson-Thompson equation; Initial boundary value problem; Galerkin method; Existence and uniqueness of global solution.

MSC: 35L20.

1. Introduction

The object of this work is to study the global solution to the following boundary value problem for the Moore-Gibson-Thompson equation

$$\alpha u_{ttt} + \beta u_{tt} - c^2\Delta u - r\Delta u_t + f(u) = 0, \text{ in } \Omega \times (0, +\infty), \tag{1}$$

$$u(x,t) = 0 \text{ on } \partial\Omega, \tag{2}$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), u_{tt}(x,0) = u_2(x), \ x \in \Omega, \tag{3}$$

where Ω is a bounded domain in $R^n(n \geq 1)$ with sufficiently smooth boundary $\partial\Omega$, $u_0(x), u_1(x)$ and $u_2(x)$ are given functions and f is a given nonlinear function. All the parameters α, β, c^2, r are assumed to be positive constants.

In recent years, increasing attention has been paid to the well-posedness and asymptotic behavior of the Moore-Gibson-Thompson (MGT) equation, see [1–7]. The MGT model is considered through third-order (in time), strictly hyperbolic partial differential equation as follows

$$\alpha u_{ttt} + \beta u_{tt} - c^2\Delta u - r\Delta u_t = f(x), \tag{4}$$

it is one of the nonlinear acoustic models describing the propagation of acoustics wave in gases and liquid, it has a wide range of applications in medical and industry. In the physical context of the acoustic waves, u is the velocity potential of the acoustic phenomena, α denotes the thermal relaxation time, c denotes the speed of sound, β denotes friction, and b denotes a parameter of diffusivity.

It is often convenient to write MGT equation as an abstract form

$$\alpha u_{ttt} + \beta u_{tt} + c^2 Au + rAu_t = f(u, u_t, u_{tt}), \tag{5}$$

and it has been shown [8,9] that the linear part of Eq. (5) generates a strongly continuous semigroup as long as $r > 0$. In [10], the authors provided a brief overview of well-posedness results, both local and global, pertinent to various configurations of MGT equations. Especially, the authors in [11] considered the following model with nonlinear control feedback

$$\tau u_{ttt} + \alpha\beta u_{tt} + c^2 Au + bAu_t + \beta u_t^3 = 2ku_t^2 + p(u), \tag{6}$$

where the parameter $\beta > 0$, $p(u)$ denotes an active force and the operator A is strictly positive. By semigroup method, it was proved in [11] we that (6) with initial data of arbitrary size in H is locally and globally well-posed under the following assumption: $p \in C^1(R)$ and its derivative satisfies $-\delta \le p'(s) \le m$ for some positive constants δ and m. Kaltenbacher *et al.*, [12] established the well-posedness by Galerkin approximations and then employ fixed-point arguments for well-posedness of the Jordan-Moore-Gibson-Thompson (JMGT) equation

$$\alpha u_{ttt} + \beta u_{tt} - b\Delta u_t - c^2\Delta u = (\frac{1}{c^2}\frac{B}{2A}u_t^2 + |\nabla u|^2)_t. \tag{7}$$

More recently, Boulaaras *et al.*, [13] proved the existence and uniqueness of the weak solution of the Moore-Gibson-Thompson equation with the integral condition by applying the Galerkin method.

In this paper, we extend the results in [11] to Problem (1)-(3) by applying the Galerkin method and compact method. The contents of this paper are organized as follows; In §2, we prepare some materials needed for our proof. Finally, in §3, we give the main result and the proof.

2. Preliminaries

Throughout this paper, the domain Ω is assumed to be sufficiently smooth to admit integration by parts and second-order elliptic regularity. We use C to denote a universal positive constant that may have different values in different places. $W^{m,2}(\Omega) = H^m(\Omega)$ and $W_0^{m,2}(\Omega) = H_0^m(\Omega)$ denote the well-known Soblev space. We denote by $||.||_p$ the $L^p(\Omega)$ norm and by $||\nabla.||$ the norm in $H_0^1(\Omega)$. In particular, we denote $||.|| = ||.||_2$

By a weak solution $u(x,t)$ of Problem (1)-(3) on $\Omega \times [0,T]$ for any $T > 0$, we mean $u \in L^\infty((0,T); H^2(\Omega) \cap H_0^1(\Omega)) \cap W^{1,\infty}((0,T); H_0^1(\Omega)) \cap W^{2,\infty}((0,T); L^2(\Omega))$, $\Delta u_t, u_{ttt} \in L^\infty((0,T); H^{-1}(\Omega))$ such that $u(x,0) = u_0(x)$ a.e. in Ω, $u_t(x,0) = u_1(x)$ a.e. in Ω, $u_{tt}(x,0) = u_2(x)$ a.e. in Ω, and

$$\alpha(u_{ttt} + \beta u_{tt} - c^2\Delta u - r\Delta u_t + f(u), v) = 0$$

for any $v \in H_0^1(\Omega)$, a.e. $t \in [0,T]$.

In this paper, we assume $\alpha, \beta, c^2, r > 0$ and

$$f \in C^1 \text{ and } |f'(s)| \le C_1. \tag{8}$$

Lemma 1. [14] *Let $\Omega \in R^n$ be a bounded domain and w_j be a base of $L^2(\Omega)$. Then for any $\epsilon > 0$ there exist a positive constant N_ϵ, such that*

$$||u|| \le (\sum_{j=1}^{N_\epsilon}(u, w_j))^{\frac{1}{2}} + \epsilon||u||_{1,p}$$

for any $u \in W_0^{1,p}(\Omega)(2 \le p < \infty)$, where N_ϵ is independent on u.

Lemma 2. [15] *Let $G(z_1, z_2, ...z_h)$ be the function of the variables $z_1, z_2, ...z_h$ and suppose that G is continuous differentiable for k-times ($k \ge 1$) with respect to every variable. Let $z_i(x,t) \in L^\infty([0,T]; H^k(\Omega))(i = 1,2,...h)$, then the estimation*

$$\int_\Omega |D_x^k G(z_1(x,t), z_2(x,t), ..., z_h(x,t))|^2 dx < C(M,k,h)\sum_{i=1}^{h}||z_i||_{H^k(\Omega)}$$

holds, where $D_x = \frac{\partial}{\partial x}$, $M = \max_{i=1,2,...,h}\max_{0\le t\le T, x\in\Omega}|z_i(x,t)|$.

3. Solvability of the problem

In this section, by using Galerkin's method and compactness method, we shall prove the existence of global solutions of Problem (1)-(3).

Let $\{w_j(x)\}_{j\in N}$ be the eigenfunctions of the following boundary problem

$$-\Delta w = \lambda w, x \in \Omega; w = 0, x \in \partial\Omega, \tag{9}$$

corresponding to the eigenvalue $\lambda_j (j = 1, 2, 3, ...)$. Then $\{w_j(x)\}_{j\in N}$ can be normalized to from an orthogonal basis of $H^2(\Omega) \cap H^1_0(\Omega)$ and to be orthnormal with respect to the $L^2(\Omega)$ scalar product.

Now, we seek an approximate solution of Problem (1)-(3) in the form of

$$u^N(x,t) = \sum_{j=1}^{N} T_{jN}(t) w_j(x), \tag{10}$$

where the constants T_{jN} are defined by the conditions $T_{jN}(t) = (u^N(x,t), w_j(x))$ and can be determined from the relation

$$\alpha(u^N_{ttt}, w_j) + \beta(u^N_{tt}, w_j) - c^2(\Delta u^N, w_j) - r(\Delta u^N_t, w_j) + (f(u^N), w_j) = 0, \tag{11}$$

$$(u^N(0), w_j) = (u_0, w_j) = u_{0j}, (u^N_t(0), w_j) = (u_1, w_j) = u_{1j}, (u^N_{tt}(0), w_j) = (u_2, w_j) = u_{2j}. \tag{12}$$

Lemma 3. *Assume (8) holds, $u_0 \in H^2(\Omega) \cap H^1_0(\Omega)$, $u_1 \in H^1_0(\Omega)$, and $u_2 \in L^2(\Omega)$, then for any $T > 0$, Problem (11)-(12) possesses a solution u^N on $[0, T]$, and the following estimate holds in the class*

$$||u^N||^2 + ||u^N_t||^2 + ||\nabla u^N||^2 + \alpha||u^N_{tt}||^2 + r||\nabla u^N_t||^2 + \int_0^t ||\nabla u^N_t||^2 d\tau + \beta||\nabla u^N_{tt}||^2 d\tau \le C. \tag{13}$$

Proof. Problem (11)-(12) leads to a system of ODEs for unknown functions $T_{jN}(t)$. Based on standard existence theory for ODE, one can obtain functions $T_{jN}(t) : [0, t_k) \to R, j = 1, 2, ..., k$, which satisfy approximate Problem (11)-(12) in a maximal interval $[0, t_k), t_k \in (0, T]$. This solution is then extended to the closed interval $[0, T]$ by using the estimate below.

Multiplying (11) by $T_{jNtt}(t)$, summing up the products for $j = 1, 2, ..., N$ and integrating by parts, we get

$$\alpha(u^N_{ttt}, u^N_{tt}) + \beta(u^N_{tt}, u^N_{tt}) + c^2(\nabla u^N, \nabla u^N_{tt}) + r(\nabla u^N_t, \nabla u^N_{tt}) + (f(u^N), u^N_{tt}) = 0. \tag{14}$$

Integrating (14) with respect to t from 0 to t, we obtain

$$\begin{aligned}\alpha||u^N_{tt}||^2 + 2\beta \int_0^t ||u^N_{tt}||^2 d\tau + r||\nabla u^N_t||^2 + 2\int_0^t (f(u^N), u^N_{tt}) d\tau \\ = -c^2 \int_0^t (\nabla u^N, \nabla u^N_{tt}) d\tau + \alpha||u^N_{tt}(0)||^2 + r||\nabla u^N_t(0)||^2.\end{aligned} \tag{15}$$

We observe that

$$\int_0^t (f(u^N), u^N_{tt}) d\tau = (f(u^N), u^N_{tt})|^t_0 - \int_0^t \int_\Omega f'(u^N)(u^N_t)^2 dx d\tau \tag{16}$$

and

$$\int_0^t (\nabla u^N, \nabla u^N_{tt}) d\tau = (\nabla u^N, \nabla u^N_t)|^t_0 - \int_0^t ||\nabla u^N_t||^2 d\tau. \tag{17}$$

Adding $2[(u^N, u^N_t) + (u^N_t, u^N_{tt}) + (\nabla u^N, \nabla u^N_t)]$ to both sides of (15) and a substitution of the equalities (16) and (17) in (15) gives

$$\begin{aligned}&\frac{d}{dt}[||u^N||^2 + ||u^N_t||^2 + ||\nabla u^N||^2] + \alpha||u^N_{tt}||^2 + 2\beta \int_0^t ||u^N_{tt}||^2 d\tau + r||\nabla u^N_t||^2 \\ &= 2[(u^N, u^N_t) + (u^N_t, u^N_{tt}) + (\nabla u^N, \nabla u^N_t)] + \alpha||u^N_{tt}(0)||^2 + r||\nabla u^N_t(0)||^2 - 2(f(u^N), u^N_{tt})|^t_0 \\ &+ 2\int_0^t \int_\Omega f'(u^N)(u^N_t)^2 dx d\tau - 2c^2(\nabla u^N, \nabla u^N_t)|^t_0 + 2c^2 \int_0^t ||\nabla u^N_t||^2 d\tau.\end{aligned} \tag{18}$$

Then, by Hölder inequality and the fact $|f(s)| = |\int_0^t f'(s) ds| \le C_1|s|$ by (A1), we arrive at

$$\begin{aligned}
&\frac{d}{dt}[||u^N||^2+||u_t^N||^2+||\nabla u^N||^2]+\alpha||u_{tt}^N||^2+2\beta\int_0^t||u_{tt}^N||^2d\tau+r||\nabla u_t^N||^2\\
&\quad\leq 2||u^N||||u_t^N||+2||u_t^N||||u_{tt}^N||+2||\nabla u^N||||\nabla u_t^N||+\alpha||u_{tt}^N(0)||^2+r||\nabla u_t^N(0)||^2\\
&\qquad+2C_1||u^N||||u_t^N||+2C_1||u^N(0)||||u_t^N(0)||+2C_1\int_0^t||u_t^N||^2d\tau\\
&\qquad+2c^2||\nabla u^N||||\nabla u_t^N||+2c^2||\nabla u^N(0)||||\nabla u_t^N(0)||+2c^2\int_0^t||\nabla u_t^N||^2d\tau\\
&\quad\leq\frac{1}{2}(\alpha||u_{tt}^N||^2+r||\nabla u_t^N||^2)+C_2(||u^N||^2+||u_t^N||^2+||\nabla u^N||^2)\\
&\qquad+2C_1\int_0^t||u_t^N||^2d\tau+2c^2\int_0^t||\nabla u_t^N||^2d\tau+\alpha||u_{tt}^N(0)||^2+r||\nabla u_t^N(0)||^2\\
&\qquad+C_3||u^N(0)||^2+C_4||u_t^N(0)||^2+c^2||\nabla u^N(0)||^2+c^2||\nabla u_t^N(0)||^2. \qquad (19)
\end{aligned}$$

Taking into account that

$$||u_{tt}^N(0)||^2+||\nabla u_t^N(0)||^2+||\nabla u^N(0)||^2\to||u_2||^2+||\nabla u_0||^2+||\nabla u_1||^2$$

and

$$||u^N(0)||^2+||u_t^N(0)||^2\to||u_0||^2+||u_1||^2$$

as $N\to\infty$, then applying the Gronwall inequality to (19) and then integrating from 0 to t appears that

$$||u^N||^2+||u_t^N||^2+||\nabla u^N||^2+\alpha||u_{tt}^N||^2+r||\nabla u_t^N||^2+\int_0^t||\nabla u_t^N||^2d\tau+\beta||\nabla u_{tt}^N||^2d\tau\leq C. \qquad (20)$$

Multiplying (11) by $\lambda_j T_{jN}(t)$ summing up the products for $j=1,2,...N$, integrating by parts and integrating with respect to t, we get

$$r||\Delta u^N||^2+c^2\int_0^t||\Delta u^N||^2d\tau=2\alpha\int_0^t(u_{ttt}^N,\Delta u^N)d\tau+2\beta\int_0^t(u_{tt}^N,\Delta u^N)d\tau+\int_0^t(f(u^N),\Delta u^N)d\tau+r||\Delta u^N(0)||^2. \qquad (21)$$

Combining Cauchy inequality, the fact $||\Delta u^N(0)||^2\to||\Delta u_0||^2$, and $|f(s)|\leq C_1|s|$, and making use of the following inequality

$$\begin{aligned}
\int_0^t(u_{ttt}^N,\Delta u^N)d\tau&=(u_{tt}^N,\Delta u^N)|_0^t-\int_0^t(u_{tt}^N,\Delta u_t^N)d\tau\\
&=(u_{tt}^N,\Delta u^N)-(u_{tt}^N(0),\Delta u^N(0))+\frac{1}{2}||\nabla u_t^N||^2-\frac{1}{2}||\nabla u_t^N(0)||^2,
\end{aligned}$$

we have

$$\begin{aligned}
&r||\Delta u^N||^2+c^2\int_0^t||\Delta u^N||^2d\tau\\
&\quad\leq 2\alpha||u_{tt}^N||||\Delta u^N||+2\alpha||u_{tt}^N(0)||||\Delta u^N(0)||+\alpha(||\nabla u_t^N||^2-||\nabla u_t^N(0)||^2)+2\beta\int_0^t||u_{tt}^N||||\Delta u^N||d\tau\\
&\qquad+\int_0^t||f(u^N)||||\Delta u^N||d\tau+r||\Delta u^N(0)||^2\\
&\quad\leq\epsilon_1||\Delta u^N||^2+C_6(||u_{tt}^N||^2+||\nabla u_t^N||^2)+C_7(||u_{tt}^N(0)||^2+||\Delta u^N(0)||^2+||\nabla u_t^N(0)||^2)\\
&\qquad+\epsilon_1\int_0^t||\Delta u^N||^2d\tau+C_8\int_0^t||u_{tt}^N||^2d\tau+C_9\int_0^t||u^N||^2d\tau. \qquad (22)
\end{aligned}$$

Choosing ϵ_1 sufficiently small and ϵ_2 sufficiently large such that $\epsilon_2>2c^2$, then it follows from (22) and (20) that

$$||\Delta u^N||^2 \leq C_{10} \int_0^t ||\Delta u^N||^2 d\tau + C_{11}. \tag{23}$$

Thus, applying Gronwall's inequality to (23), we deduce

$$||\Delta u^N||^2 \leq C. \tag{24}$$

Combining (20) and (24), we get

$$||u^N||^2 + ||u_t^N||^2 + ||\nabla u^N||^2 + ||u_{tt}^N||^2 + ||\nabla u_t^N||^2 + \int_0^t ||\nabla u_t^N||^2 d\tau + \beta ||\nabla u_{tt}^N||^2 d\tau \leq C. \tag{25}$$

Furthermore, by (25), we have that (11)-(12) possesses a global solution. □

Theorem 1. *Assume (8) holds, $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$, $u_1 \in H_0^1(\Omega)$, and $u_2 \in L^2(\Omega)$, then for any $T > 0$, Problem (1)-(3) possesses a unique global solution.*

Proof. For any $v \in H_0^1(\Omega)$, it follows that

$$\alpha |(u_{ttt}^N, v)| \leq (\beta ||u_{tt}^N|| + c^2 ||\Delta u^N|| + ||\nabla u_t^N|| + C_1 ||u^N||)||v||_{H_0^1}. \tag{26}$$

Thus, using Lemma 3, it follows that

$$||u_{ttt}^N||_{H^{-1}(\Omega)} \leq M. \tag{27}$$

Similarly, we have

$$||\Delta u_t^N||_{H^{-1}(\Omega)} \leq M. \tag{28}$$

From Lemma 3, (27) and (28), there exist a subsequence of $\{u^N\}$, still denoted by $\{u^N\}$, and a function u, ξ, η, such that

$$u^N \to u \text{ weak} * \text{ in } L^\infty(0,T,H^2(\Omega) \cap H_0^1(\Omega)), \tag{29}$$
$$u_t^N \to u_t \text{ weak} * \text{ in } L^\infty(0,T,H_0^1(\Omega)), \tag{30}$$
$$u_{tt}^N \to u_{tt} \text{ weak} * \text{ in } L^\infty(0,T,L^2(\Omega)), \tag{31}$$
$$u_{ttt}^N \to u_{ttt} \text{ weak} * \text{ in } L^\infty(0,T,H^{-1}(\Omega)), \tag{32}$$
$$f(u^N) \to \xi \text{ weak} * \text{ in } L^\infty(0,T,H^{-1}(\Omega)), \tag{33}$$
$$\Delta u_t^N \to \eta \text{ weak} * \text{ in } L^\infty(0,T,H^{-1}(\Omega)). \tag{34}$$

and for any $t \in [0, T]$

$$u^N \to u \text{ weakly in } H^2(\Omega) \cap H_0^1(\Omega), \tag{35}$$
$$u_t^N \to u_t \text{ weakly in } H_0^1(\Omega), \tag{36}$$
$$u_{tt}^N \to u_{tt} \text{ weakly in } L^2(\Omega), \tag{37}$$
$$u_{ttt}^N \to u_{ttt} \text{ weakly in } H^{-1}(\Omega), \tag{38}$$
$$f(u^N) \to \xi \text{ weak} * \text{ in } H^{-1}(\Omega)), \tag{39}$$
$$\Delta u_t^N \to \eta \text{ weak} * \text{ in } H^{-1}(\Omega)). \tag{40}$$

Since $f \in C^1$ and $||f(u^N)|| \leq C||u^N|| \leq C$, for any $v \in H_0^1(\Omega)$ and any $t \in [0, T]$, we have

$$(\Delta u_t^N, v) = -(\nabla u_t^N, \nabla v) \to -(\nabla u_t, \nabla v) = (\Delta u_t, v), \tag{41}$$
$$f(u^N) \to f(u) \tag{42}$$

as $N \to \infty$. Then we get $\xi = f(u), \eta = \nabla u_t$, combining this with (35)-(40), we have

$$u \in L^\infty((0,T); H^2(\Omega) \cap H_0^1(\Omega)) \cap W^{1,\infty}((0,T); H_0^1(\Omega)) \cap W^{2,\infty}((0,T); L^2(\Omega)),$$
$$\Delta u_t, u_{ttt} \in L^\infty((0,T); H^{-1}(\Omega)).$$

By using Lemma 3 and (27), we observe that

$$|(u^N, w_j)| + \sum_{k=1}^{3} |(u^N_{t^k}, w_j)| \leq M, \tag{43}$$

where $u^N_{t^k} = \frac{\partial^k u^N}{\partial t^k}$. Then, by Ascoli-Arcela theorem, we can select from $\{u^N\}$ a subsequence, still denoted by $\{u^N\}$, such that as $N \to \infty$, the subsequence

$$(u^N, w_j) \ \to \ (u, w_j),\ (u^N_{t^k}, w_j) \ \to \ (u_{t^k}, w_j),\ k = 1, 2, 3,\ j = 1, 2..... \tag{44}$$

In particular, we take $t = 0$ and we note that $\{w_j(x)\}_{j \in N}$ are an orthogonal basis of $L^2(\Omega)$, we know that

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ u_{tt}(x,0) = u_2(x)\ a.e.\ in\ \Omega. \tag{45}$$

By (29)-(34),(44) and Lemma 2.1, we have

$$u^N \ \to u,\ u^N_t \ \to \ u_t\ in\ C([0,T], L^2(\Omega)). \tag{46}$$

Thanks to (29)-(42), letting $N \to \infty$ in (11), leads to

$$\alpha(u_{ttt}, v) + \beta(u_{tt}, v) - c^2(\Delta u, v) - r(\Delta u_t, v) + (f(u), v) = 0 \tag{47}$$

for any $v \in H_0^1(\Omega)$. Altogether, we conclude that u is a solution of the initial boundary Problem (1)-(3).

Now, suppose that there exist two different solutions u_1, u_2 for Problem (1)-(3), then the difference $w = u_1 - u_2$ satisfies

$$\alpha w_{ttt} + \beta w_{tt} - c^2 \Delta w - r\Delta w_t + f(u_1) - f(u_2) = 0,\ in\ \Omega \times (0, +\infty), \tag{48}$$
$$w(x,t) = 0\ on\ \partial\Omega, \tag{49}$$
$$w(x,0) = 0, w_t(x,0) = 0, w_{tt}(x,0) = 0,\ x \in \Omega, \tag{50}$$

Integrating (48) for t from 0 to t, we have

$$\alpha w_{tt} + \beta w_t - r\Delta w = \int_0^t (c^2 \Delta w + f(u_2) - f(u_1)) d\tau. \tag{51}$$

Multiplying the Eq. (51) by w_t, integrating over Ω, adding up (w, w_t), we obtain

$$\begin{aligned}\frac{1}{2}(\alpha||w_t||^2 + r||\nabla w||^2 + ||w||^2) + \beta||w_t||^2 &= 2\int_0^t (c^2\Delta w + f(u_2) - f(u_1)) w_t d\tau \\ &= 2c^2(||\nabla w||^2 - ||\nabla w_0||^2) + 2\int_0^t \int_\Omega \theta w w_t dx d\tau \\ &\leq C(||\nabla w||^2 + ||w_t||^2),\end{aligned} \tag{52}$$

where we have used mean value theorem and $|\theta| \leq 1$. By applying Gronwall inequality, we deduce that

$$\alpha||w_t||^2 + r||\nabla w||^2 + ||w||^2 = 0. \tag{53}$$

This implies that $w = 0$ for all $t \in [0, T]$. Thus the uniqueness is proved. □

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