Open Journal of
Mathematical Analysis

PSR Press

Article

Forecasting the democratic republic of the Congo macroeconomic data with the Bayesian vector autoregressive models

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Communicated by: Absar Ul Haq

Received: 3 November 2022; Accepted: 29 November 2022; Published: 31 December 2022.

Abstract: The purpose of this paper is to emphasize the role of the Bayesian Vector Autoregressive models (VAR) in macroeconomic analysis and forecasting. To help the policy-makers to do better, the Bayesian VAR models are considered more robust and valuable because they put in the model the mathematician's beliefs or priors and the data. By using BVAR(1), we get the main results: (i) the best out sample point forecasts; (ii) the exchange rate shock contributes more to inflation; (iii) the inflation shock has high effects on exchange rate innovation. These results are due to the dollarization of this small open economy.

Keywords: Bayesian parameter estimation; Forecasting; Democratic republic of the Congo; Macroeconometric modelling; Uncertainties; Vector autoregressive processes.

MSC: 60G15; 62C10; 62H12; 62F05.

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1. Introduction

Forecasting is one of the objectives of multiple time series analysis [1]. In the considerable time series analysis, we often used the Vector Autoregressive models, so-called, in short, VAR models. They are one of the most successful statistical modeling ideas that have come up in the last forty years. The use of Bayesian methods makes the VAR models generic enough to handle a variety of complex real-world time series [1–4]. Moreover, the VAR models consider the time series' dynamic behaviors. In economics, these dynamic behavior have many interesting contributions to the development of the theory, like rational expectations, causality, correlation, persistence, the cointegration of the macroeconomic variables, the convergence of economies, etc.

Bayesian statistics provides a rational theory of personal beliefs compounded with real-world data in uncertainty. In the last three decades, Bayesian Statistics has emerged as one of the leading paradigms in which all of this can be done in a unified fashion. As a result, there has been tremendous development in Bayesian theory, methodology, computation, and applications in the past several years [5,6].

The appreciation of the potential for Bayesian methods is growing fast both inside and outside the econometrics community. The first encounter with Bayesian ideas by many people simply entails discovering that a particular Bayesian method is superior to classical statistical methods in a specific problem or question. Nothing succeeds like success, and this observed superiority often leads to the further pursuit of Bayesian analysis.

For scientists with little or no formal statistical background, Bayesian methods are being discovered as the only viable method for approaching their problems. Unfortunately, for many of them, statistics has become synonymous with Bayesian statistics.

Bayesian Vector Autoregressive models are one of the most successful statistical modelling ideas that have come up in the last four decades. The use of Bayesian methods makes the models generic enough to handle a variety of complex real-world time series.

The purpose of Bayesian inference is to provide mathematical machinery that can be used for modeling systems, where the uncertainties of the system are taken into account and the decisions are made according to rational principles [5–8].

The Bayesian method, as many might think, is not new but a method that is older than many of the commonly known and well-formulated statistical techniques. The basis for Bayesian statistics was laid down in a revolutionary paper written by British mathematician and Reverend Thomas Bayes (1702 - 1761), which appeared in print in 1763 but was not acknowledged for its significance [9–11].

The rest of this paper is organized as follows. §2 presents the Bayesian vector autoregressive model. The §3 gives the empirical results. The §4 gives the asymptic properties of Bayesian methods for stochastic differential equation models.

2. Bayesian vector autoregressive model

Vector Autoregressions (VARs) are linear multivariate time-series models that capture the joint dynamics of multiple time series. As mentioned by Tsay, the most commonly used multivariate time series model is the vector autoregressive (VAR) model, particularly in the econometric literature, for good reasons. First, the model is relatively easy to estimate. One can use the least-squares (LS), the maximum likelihood (ML), or the Bayesian method. All three estimation methods have closed-form solutions. For a VAR model, the least-squares estimates are asymptotically equivalent to the ML estimates. The ordinary least-squares (OLS) estimates are the same as the generalized least-squares (GLS) estimates. Second, the properties of VAR models have been studied extensively in the literature. Finally, VAR models are similar to the multivariate multiple linear regressions widely used in multivariate statistical analysis. Many methods for making inferences in multivariate multiple linear regression applied to the VAR model [12].

The pioneering work of Sims [13] proposed to replace the large-scale macroeconomic models popular in the 1960s with VARs and suggested that Bayesian methods could have improved upon frequentist ones in estimating the model coefficients. As a result, bayesian methods are increasingly becoming attractive to researchers in many fields, such as Econometrics [14]. Bayesian VARs (BVARs) with macroeconomic variables were first employed in forecasting by Litterman[15] and Doan, Litterman, and Sims[16]. Still, now it is one of the most macro-econometric tools routinely used by scholars and policymakers for structural analysis and scenario analysis in an ever-growing number of applications.

Suppose X_t a vector of $n \times n$ is a stationary Gaussian VAR(1) process of the form

$$X_t = \Pi X_{t-1} + U_t, \quad U_t \sim N(0_{n\times n}, \Omega_{n\times n}), \tag{1}$$

where $\Pi_{n\times n}$ denotes the matrix of coefficients, U_t is a vector of innovations of $n \times 1$ and the prior distribution for $\Theta := vec(\Pi)$ is a multivariate normal with known mean Θ^* and covariance matrix Ω_Θ. For the reasons of simplicity and practice, stationary, stable VAR(1) process has been considered. As well known in time series literature, a process is stationary if it has time invariant first and second moments. Since X_t follows a VAR(1) model, the condition for its stationarity is that all solutions of the determinant equation $\mid I_{kp} - \Phi B \mid= 0$ must be greater that 1 in modulus or they are outside the unit circle [1,2,17].

The multivariate time series X_t follows a vector autoregressive model of order p, VAR(p), that is a generalization of a vector autoregressive model of order 1, VAR(1), if

$$X_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i X_{t-i} + \epsilon_t\,, \tag{2}$$

where Φ_0 is a k-dimensional constant vector and Φ_i are $k \times k$ matrices for $i > 0$; $\Phi_p \neq 0$, and ϵ_t is a sequence of independent and identically distributed $(i.i.d.)$ random vectors with mean zero and covariance matrix Σ_ϵ, which is positive - definite.

In econometric analysis and multivariate time series analysis, the useful tools used by policymakers are the forecast error variance decomposition and the impulse response functions. The MA representation of the VAR(p) model

$$X_t = \mu + \sum_{i=1}^{\infty} \Psi_i \epsilon_{t-i}\,. \tag{3}$$

We can express a VAR(p) model in a VAR(1) form by using an expanded series. Define $Y_t = (X_t', X_{t-1}', ..., X_{t-p+1}')'$, which is pk- dimensional time series. The VAR(p) in Eq. 3 can be written as

$$Y_t = \Xi Y_{t-1} + \Lambda_t , \tag{4}$$

where $\Lambda_t = (U_t', 0')$ with 0 being a $k(p-1)$ - dimensional zero vector, and

$$\Xi = \begin{bmatrix} \psi_1 & \psi_2 & ... & \psi_{p-1} & \psi_p \\ I & 0 & ... & 0 & 0 \\ 0 & I & ... & 0 & 0 \\ .... & ... & ... & ... & ... \\ 0 & 0 & ... & I & 0 \end{bmatrix}, \tag{5}$$

where it is understood that I and 0 are the $k \times k$ identity and zero matrix, respectively. The matrix Ξ is called the companion matrix of the matrix polynomial $\Psi(B) = I_k - \Psi_1 B - ... - \Psi_p B_p$. The covariance matrix of Λ_t has a special structure; all of its elements are zero except those in the upper-left corner that is Σ_ϵ.

Also, the necessary and sufficient condition for the weak stationarity of the VAR(p) series is that all solutions of the determinant equation $\mid \Psi(B) \mid= 0$ must be greater than 1 in modulus.

2.1. Bayesian estimation methods

As mentioned in [1,3,18], in the Bayesian approach, it is assumed that the non sample or prior information is available in the form of a density. Denoting the parameters of interest by Θ, let us assume that the prior information is summarized in the prior probability density function (p.d.f.) $g(\Theta)$. The sample information is summarized in the sample, say $f(y \mid \Theta)$, which is algebraically identical to the likelihood function $\mathcal{L}(\Theta \mid X)$. The two types of information are combined via Bayes' theorem which states that

$$g(\Theta \mid X) = \frac{f(X \mid \Theta) g(\Theta)}{f(X)}, \tag{6}$$

where $f(X)$ denotes the unconditional sample density which, for a given sample, is just a normalizing constant. In other words the distribution of Θ, given the sample information contained in X, can be summarized by $g(\Theta \mid X)$. This function is proportional to the likelihood function times the prior density $g(\Theta)$,

$$g(\Theta \mid X) \propto f(X \mid \Theta) g(\Theta) = \mathcal{L}(\Theta \mid X) g(\Theta) . \tag{7}$$

The conditional density $g(\Theta \mid X)$ is the posterior p.d.f.. It contains all the information available on the parameter vector Θ. Point estimators of Θ may be derived from the posterior distribution. That is,

$$\text{posterior distribution} \propto \text{likelihood} \times \text{prior distribution}. \tag{8}$$

The normal priors for the parameters of a Gaussian VAR model, $\Theta := vec(A) = vec(A_1, ..., A_p)$ is a multivariate normal with known mean Θ^* and covariance matrix Ω_θ,

$$g(\Theta) = \Big(\frac{1}{2\pi}\Big)^{K^2 p/2} \mid \Omega_\theta \mid^{-1/2} exp\Big[-\frac{1}{2}(\Theta - \Theta^*)' \Omega_\theta^{-1} (\Theta - \Theta^*) \Big]. \tag{9}$$

The Gaussian likelihood function

$$g(\Theta) = \Big(\frac{1}{2\pi}\Big)^{KT/2} \mid I_T \otimes \Sigma_u \mid^{-1/2} exp\Big[-\frac{1}{2}(X - (Z' \otimes I_T)\Theta)' (I_T \otimes \Sigma_u^{-1})(X - (Z' \otimes I_T)\Theta) \Big]. \tag{10}$$

Combining the prior information with the sample information summarized in the Gaussian likelihood function gives the posterior density

$$g(\Theta) \propto \mathcal{L}(\Theta \mid X) g(\Theta) \propto exp\Big\{ -\frac{1}{2}[(\Omega_\theta^{-1/2}(\Theta - \Theta^*)^{'}(\Omega_\theta^{-1/2}(\Theta - \Theta^*)] + \{(I_T \otimes \Sigma_u^{-1})X - (Z^{'} \otimes \Sigma_u^{-1/2})\Theta\}^{'}\{(I_T \otimes \Sigma_u^{-1})X - (Z^{'} \otimes \Sigma_u^{-1/2})\Theta\}]\Big\}. \quad (11)$$

Here $\Omega_\theta^{-1/2}$ and $\Sigma_u^{-1/2}$ denote the symmetric square root matrices of Ω_θ^{-1} and Σ_u^{-1}, respectively. The white noise covariance matrix Σ_u is assumed to be known for the moment. Defining $w^{'} := \left[\Omega_\theta^{-1/2}\Theta^* \quad (I_T \otimes \Sigma_u^{-1})X\right]^{'}$ and $W^{'} := \left[\Omega_\theta^{-1/2} \quad Z^{'} \otimes \Sigma_u^{-1}\right]^{'}$, the exponent in (11) can be rewritten as

$$-\frac{1}{2}(w - W\Theta)^{'}(w - W\Theta) = -\frac{1}{2}\Big[(\Theta - \bar{\Theta})^{'}W^{'}W(\Theta - \bar{\Theta}) + (w - W\bar{\Theta})^{'}(w - W\bar{\Theta})\Big],$$

where

$$\bar{\Theta} := (W^{'}W)^{'}Ww = [\Omega_\theta^{-1} + (ZZ^{'} \otimes \Sigma_u^{-1})]^{-1}[\Omega_\theta^{-1}\Theta^* + (Z^{'} \otimes \Sigma_u^{-1})X]\,. \quad (12)$$

The final values of the parameters obtained in the computation are called the posterior mean of VAR(1) coefficients estimated by using Minnesota.

3. Forecasting methods

According to literature, we have many methods of forecasting. But here we are talking about two of them suck as point forecasts and interval forecasts. For point Forecast, let a general BVAR(p) process with zero mean,

$$X_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i X_{t-i} + \epsilon_t \quad (13)$$

has a BVAR(1)

$$Y_t = \Pi Y_{t-1} + U_t, \;\; U_t \sim N(0_{n\times n}, \Omega_{n\times n}), \quad (14)$$

the optimal predictor of Y_{t+h} can be seen

$$Y_t(h) = \Pi^h Y_{t-1} = \Pi Y_t(h-1). \quad (15)$$

For interval forecasts, assume the BVAR process and the forecast errors are the Gaussian processes. A $(1-\alpha)100\%$, interval forecasts, h periods ahead, for the $k-th$ component of y_t is

$$X_{k,t}(h) \pm z_{(\alpha/2)}\sigma_k\,, \quad (16)$$

where σ_k is the square root of the $k-th$ diagonal element of Ω_ε. Fr example, if the distribution is normal for 95% of confidence, statistic Z - Score equals to 1,96, that is, $z_{(\alpha/2)} = 1.96$.

3.1. Forecast error variance decomposition

As presented in [17], by using the MA representation of a VAR(p) model in Equation 0000 and the fact that $Cov(\eta_t) = I_k$, we see that the $l-step$ ahead error of Z_{h+l} at the forecast origin $t = h$ can be written as

$$e_h(l) = \psi_0\eta_{h+l} + \psi_1\eta_{h+l-1} + ... + \psi_{l-1}\eta_{h+l}, \quad (17)$$

and the covariance matrix of the forecast error is

$$Cov[e_h(l)] = \sum_{v=0}^{l-1} \psi_v\psi_v^{'}\,. \quad (18)$$

From Eq. (18), the variance of the forecast error $e_{h,i}(l)$, which is the *ith* component of $e_h(l)$ is

$$Var[e_{h,i}(l)] = \sum_{v=0}^{l-1}\sum_{j=1}^{k}\psi_{v,ij}^2 = \sum_{j=1}^{k}\sum_{v=0}^{l-1}\psi_{v,ij}^2 . \tag{19}$$

Using Eq. (19), we define

$$\omega_{ij}(l) = \sum_{v=0}^{l-1}\psi_{v,ij}^2, \tag{20}$$

and obtain

$$Var[e_{h,i}(l)] = \sum_{j=1}^{k}\omega_{ij}(l). \tag{21}$$

Therefore, the quantity $w_{ij}(l)$ can be interpreted as the contribution of the jth shock η_{jt} to the variance of the $l - step$ ahead forecast error of Z_{it}. Eq. (21) is referred to as the forecast error decomposition. In particular, $\omega_{ij}(l)/Var[e_{h,i}(l)]$ is the percentage of contribution from the shock η_{jt}.

3.2. Impulse response functions

Suppose that the bivariate time series z_t consists of monthly inflation and exchange rate growth; one might be interested in knowing the effect on the monthly inflation rate if the monthly exchange rate growth is increased or decreased by one. This type of study is referred to as the impulse response function in the statistical literature and the multiplier analysis in the econometric literature.

The coefficient matrix Ψ_i of the MA representation of a VAR(p) model referred to as the coefficients of impulse response functions. The summation $\Phi_n = \sum_{i=0}^{n}\Psi_i$ denotes the accumulated responses over n periods to a unit shock to Z_t. From the MA representation of Z_t and using the Cholesky decomposition of Σ_ε, we have

$$Z_t = [\Phi_0 + \Phi_1 B + \Phi_2 B^2 + ...]\eta_t , \tag{22}$$

where $\Phi_l = \Psi_l U'$, $\Sigma_\varepsilon = U'U$, and $\eta_t = (U')^{-1}\varepsilon_t$ for $l \geq 0$. Thus, components of η_t are uncorrelated and have unit variance. The total accumulated responses for all future periods are defined as $\Phi_\infty = \sum_{i=0}^{\infty}\Psi_i$ and called the total multipliers or long - run effects.

4. Empirical results

The Bayesian vector autoregressive models are often used in any scientific field where forecasting is used to lead the policy analysis. For example, the BVAR models are considered must macroeconometric analysis tools in macroeconomics.

4.1. Economic intuitions behind the VAR(1) model

Macroeconometric modeling increased in importance during the late I950S and the I960s, achieving a very influential role in macroeconomic policy-making during the I970s. Its failure to deliver the precise economic control it had seemed to promise then led to a barrage of attacks, ranging from disillusion and skepticism on the part of policymakers to detailed and well-argued academic criticism of the basic methodology of the approach.

The most powerful and influential of these academic arguments came from Sims (I980) in his article 'Macroeconomics and Reality. On three quite separate grounds, Sims argued against the basic process of model identification, which lies at the heart of the Cowles Commission methodology. First, most economists would agree that there are many macroeconomic variables whose cyclical fluctuations are of interest and would agree further that fluctuations in these series are interrelated [13]. Thus, the weakness of BVAR models is these models do not have little any theory. In time series analysis or econometrics, they are well-known as "Black-Box models."

4.2. Data analysis

To illustrate the Bayesian VAR(1) model using some informative priors such as Minnesota. We use monthly data from the Democratic Republic of the Congo data set on inflation rate π_t, the change of exchange rate e_t, money growth m_t, and the evolution of the cooper price h_t. The sample runs from January 2004 to September 2018. These four variables are commonly used in the D.R.C's forecasting. We put copper prices here because the mining sector dominates this economy. The D.R.C. is the fourth Copper producer country in the World. Therefore, any change in price in the international market affects positively or negatively the macroeconomic stability. This summary statistics is given in Table 1.

Table 1. Summary statistics

	π_t	e_t	m_t	h_t
Mean	0.0126	0.0083	0.0191	0.0052
Median	0.0055	-0.0022	0.0161	0.0074
Max	0.1139	0.1054	0.1758	0.2308
Min	-0.0746	-0.0970	-0.1177	-0.3501
Std Dev	0.0206	0.0248	0.0521	0.0719
Skewness	1.5536	0.6023	0.2024	-0.7503
Kurtosis	9.3533	6.8723	3.1228	7.7440
Jarque - Bera Stat	366.8031	120.6023	1.3117	181.5513
Prob (JB)	0.000	0.0000	0.5190	0.0000
Sum	2.2170	1.4601	3.3662	0.9084
Observations	176	176	176	176

Using the software Eviews 11 and the data, the maximum lag of BVAR model is 1. Therefore, the forecasting and structural analysis will be done with the BVAR(1) model. These are the popular information criterion for the selection of the model. AIC: Akaike information Criterion, SC: Schwarz Information Criterion, HQ: Hannan - Quinn Information Criterion, and JB: Jarque - Bera. According to these empirical results, the optimal lag of our model is 1. Therefore, we will estimate the VAR(1)process using the Bayesian estimation method. The selection of lag lengh is presented in Table 2.

Table 2. Selection of lag lengh

Lag	log L	AIC	SC	HQ
0	1280.9	-15.20	-15.13	-15.17
1	1370.7	-16.08^*	-15.71^*	-15.93^*
2	1379.7	-16.00	-15.33	-15.72
3	1387.67	-15.90	-14.93	-15.50
4	1394.73	-15.79	-14.53	-15.28

(*) indicates the calculated optimal lag of the VAR(p) model.

4.3. Estimated posterior mean coefficients of VAR

To illustrate this approach in simple way, our Bayesian VAR(1) model takes this matrix form,

$$\begin{bmatrix} \pi_t \\ e_t \\ m_t \\ h_t \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \end{bmatrix} \times \begin{bmatrix} \pi_{t-1} \\ e_{t-1} \\ m_{t-1} \\ h_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}, \tag{23}$$

where π_t, m_t,e_t, and h_t denote the monthly CPI inflation rate, the change of exchange rate, money growth, and the change of the cooper price, respectively. $\{u_{it}, i = 1, 2, 3, 4\}$ denote the shock of the economic polices. In literature, they are so-called the "innovations". Assume u_{it} are the Gaussian white processes.

Table 3. BVAR coefficients for litterman/minnesota prior

	π_t	e_t	m_t	h_t
π_{t-1}	0.3168	0.1597	0.3103	0.1643
	[5.6671]	[2.3529]	[1.9373]	[0.8613]
e_{t-1}	0.2863	0.2593	0.0682	0.2301
	[5.7806]	[4.2812]	[0.4793]	[1.3582]
m_{t-1}	0.0616	0.0794	-0.1410	0.0248
	[2.9656]	[3.1424]	[-2.3543]	[0.3494]
h_{t-1}	0.0167	-0.0451	-0.0428	0.2792
	[1.0128]	[-2.2471]	[-0.9030]	[4.9282]

With VARs, the parameters themselves are rarely of direct interest because the fact that there are so many of them makes it hard for the reader to interpret the table of VAR coefficients. However, Table 3 presents posterior means of all the VAR coefficients for Minnesota denotes student statistics used for the statistical significance of parameters.

4.4. The congolese macroeconomic forecasts

The methods of Forecasting and nowcasting the economy are risky, often humbling tasks. But, unfortunately, they are the jobs that many statisticians, economists, and others are required to engage in as mentioned in many papers [15,18]. Nowadays, VARs models have become powerful Forecasting tools in many Central Banks and other intuitions. The outputs from Bayesian VAR models seem more accurate and robust than those of different approaches. The congolese macroeconomic forecasts from October 2018 to March 2019 is given in Table 4.

Table 4. The congolese macroeconomic forecasts: Oct. 2018 - March 2019

	Inflation	Exchange rate	money growth	Cooper price
October2018	0.0126	0.0083	0.0191	0.0052
November 2018	0.0055	-0.0022	0.0161	0.0074
December 2018	0.1139	0.1054	0.1758	0.2308
January 2019	-0.0746	-0.0970	-0.1177	-0.3501
February 2019	0.0206	0.0248	0.0521	0.0719
March 2019	1.5536	0.6023	0.2024	-0.7503

4.5. Forecast error variance decompositions

In time series analysis, the Bayesian VAR models attract the interest of many researchers in all real-world fields such as Economics, Finance, Geoscience, Physics, Biology, etc. [9,17–19]. Indeed, the VAR models are used not only for forecasting and nowcasting but also as policy analysis tools by using impulse response functions and variance decomposition [13,20].

The variance decomposition of inflation shows that 82 % of its innovations are due to itself innovations, and 13 %, 6 %, and 41 points of the percentage are due to exchange rate, money, and cooper price index innovations. With 13 % the exchange rate contributes more to the CPI inflation.

The variance decomposition of the exchange rate shows that 79 % of its innovations are due to itself innovations and 15 %, 3 %, and 2 points of the percentage are due to inflation rate, money, and cooper price index innovations.

The variance decomposition of money shows that 97 % of its innovations are due to itself innovations and 2 % , 1 % , and 27 points of percentage are due to inflation, exchange rate, and cooper price index innovations.

For policy-makers, these results show a close connexion between the inflation rate and exchange rate because of this economy's dollarization and the economy's extraversion. This monetary phenomenon dated since the 1990s, when the Congolese economy fell down by the political and socio-economic crises and army conflicts of the early 2000s.

5. Conclusion

This study presents the Bayesian vector autoregressive models and applies them to forecast the D.R.C.'s macroeconomic data. The Bayesian vector autoregressive models are intensively used in the macro econometric analysis to highlight the policy-making process by improving the structural analysis and the forecasts. The Bayesian vector autoregressive models are thoroughly more used in macroeconomics because they can put together uncertainties inherent to real-world economic problems. In this work, first, we give the mathematical and statistical foundations of the BVAR models, and last, we use these models for policy-making. We get two main results using BVAR model tools and macroeconomic data. First, there is a close relationship between the inflation rate and exchange rate change in the Democratic Republic of the Congo. The use of U.S. money can explain this result, that is, the dollar in the different transactions inside the country's so-called "dollarization" economy, and the D.R.C.'s economy is a small and open economy. Secondly, the weak effects of money growth in inflation. This result is close to the paradigm of "money neutrality in the short run." This means that in the short term, money growth does not affect inflation. According to the Quantity Theory of Money (Q.T.M.), the changes in the price level are related to the change in money. Thirdly, the copper price shocks affect more exchange rate. And finally, the BVAR models give the best forecasts among other V.A.R. models.

Acknowledgments: The authors would like to thank the referee for his/her valuable comments that resulted in the present improved version of the article.

Author Contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflicts of Interest: "The authors declare no conflict of interest."

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