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# A METHOD TO COMPUTE THE DETERMINANT OF A $5 \times 5$ MATRIX

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Abstract. In this paper we have presented a new method to compute the determinant of a $5 \times 5$ matrix.

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*Key words and phrases:* Determinant; $5 \times 5$ matrix; Condensation.

## 1. Introduction

The determinant of an $n \times n$ matrix

$$A_n = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n\times n}$$

is denoted by $det(A_n)$ or $|A_n|$, and a basic formula to compute the determinant is

$$det(A_n) = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = \sum sgn(j_1, j_2, ..., j_n) a_{1j_1} a_{2j_2} ... a_{nj_n},$$

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where the summation is taken over all $n$ permutations $j_1, j_2, ..., j_n$ of the set of integers $1, 2, ..., n$. Furthermore, the function $sgn(j_1, j_2, ..., j_n)$ is defined as:

$$sgn(j_1, j_2, , j_n) = \begin{cases} +1 & \text{if } j_1, j_2, ..., j_n \text{ is an even permutation,} \\ -1 & \text{if } j_1, j_2, ..., j_n \text{ is an odd permutation.} \end{cases}$$

In this paper, we will present a new method to compute the determinant of a $5 \times 5$ matrix.

## 2. Preliminaries: The main definitions and lemmas

In matrix theory, a square matrix is called *nonsingular* if and only if its determinant is nonzero. We generalize the nonsingular matrices to *doubly nonsingular*:

**Definition 2.1.** An $n \times n$ matrix $A_n = [a_{ij}]_{n\times n}$ is doubly nonsingular if and only if $A_n$ is nonsingular and all $2 \times 2$ matrices of adjacent terms within the $A_n$ are nonsingular.

**Example 2.2.** Let $A_3 = \begin{bmatrix} 1 & 6 & 3 \\ 3 & 2 & 2 \\ 6 & 1 & 3 \end{bmatrix}_{3\times 3}$. We have $det(A_3) = -5$, consequently $A_3$ is nonsingular. Clearly, all $2 \times 2$ determinants of adjacent terms are nonzero. Hence, the matrix $A_3$ is doubly nonsingular.

Now, we introduce a new function, which we call the *star fraction*:

**Definition 2.3.** Given two $3 \times 3$ matrices $A_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$ and $B_3 = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times 3}$ such that $b_{ij} \neq 0 (\forall i, j = 1, 2, 3)$ and $B_3$ is doubly nonsingular. The star fraction of $A_3$ on $B_3$ is defined as:

$$\left(\frac{A_3}{B_3}\right)^* = \left(\frac{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times 3}}\right)^* = \frac{\begin{vmatrix} \frac{\begin{vmatrix} \frac{a_{11}}{b_{11}} & \frac{a_{12}}{b_{12}} \\ \frac{a_{21}}{b_{21}} & \frac{a_{22}}{b_{22}} \end{vmatrix}}{\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}} & \frac{\begin{vmatrix} \frac{a_{12}}{b_{12}} & \frac{a_{13}}{b_{13}} \\ \frac{a_{22}}{b_{22}} & \frac{a_{23}}{b_{23}} \end{vmatrix}}{\begin{vmatrix} b_{12} & b_{13} \\ b_{22} & b_{23} \end{vmatrix}} \\ \frac{\begin{vmatrix} \frac{a_{21}}{b_{21}} & \frac{a_{22}}{b_{22}} \\ \frac{a_{31}}{b_{31}} & \frac{a_{32}}{b_{32}} \end{vmatrix}}{\begin{vmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix}} & \frac{\begin{vmatrix} \frac{a_{22}}{b_{22}} & \frac{a_{23}}{b_{23}} \\ \frac{a_{32}}{b_{32}} & \frac{a_{33}}{b_{33}} \end{vmatrix}}{\begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix}} \end{vmatrix}}{\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}}.$$

In the next section, we will show that the star fraction is a useful function for calculating the determinant of a $5 \times 5$ matrix.

Now, we shall know about the *Dodgson condensation* of a matrix that was introduced by Charles Lutwidge Dodgson in 1866 [1]:

**Definition 2.4.** The Dodgson condensation of an $n \times n$ matrix $A_n = [a_{ij}]_{n\times n}$ is an $(n-1) \times (n-1)$ matrix such as $[m_{ij}]_{(n-1)\times(n-1)}$ such that

$$m_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}.$$

Henceforth the notation $DC(A_n)$ is denote the first Dodgson condensation of a matrix $A_n$, and the second condensation is $DC(DC(A_n))$ and so on. Clearly a square matrix $A_n$ is doubly nonsingular if and only if all elements of $DC(A_n)$ are nonzero.

To prove the main theorem we need the following lemmas:

**Lemma 2.5.** [3] *We have*

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} = \frac{\begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} & a_{14} & a_{15} \\ a_{22} & a_{23} & a_{24} & a_{25} \\ a_{32} & a_{33} & a_{34} & a_{35} \\ a_{42} & a_{43} & a_{44} & a_{45} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} & a_{24} & a_{25} \\ a_{32} & a_{33} & a_{34} & a_{35} \\ a_{42} & a_{43} & a_{44} & a_{45} \\ a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}},$$

*where* $\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} \neq 0.$

**Lemma 2.6.** [2, Theorem 1] *We have*

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = \frac{\begin{vmatrix} \dfrac{\begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \end{vmatrix}}{a_{22}} & \dfrac{\begin{vmatrix} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{vmatrix} \\ \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} \end{vmatrix}}{a_{23}} \\ \dfrac{\begin{vmatrix} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \\ \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix} & \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} \end{vmatrix}}{a_{32}} & \dfrac{\begin{vmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} \\ \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} & \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \end{vmatrix}}{a_{33}} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}},$$

*where* $a_{22}, a_{23}, a_{32}, a_{33}$ *are nonzero numbers and* $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \neq 0$.

## 3. Main results

In the following theorem we establish a new method to compute the determinant of a $5 \times 5$ matrix.

**Theorem 3.1.** *Given a* $5 \times 5$ *matrix*

$$A_5 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}_{5\times 5},$$

*where* $\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}$ *is a doubly nonsingular matrix with all nonzero elements. Then*

$$det(A_5) = \left( \frac{DC(DC(A_5))}{\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}_{3\times 3}} \right)^* .$$

*Proof.* Since $\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} \neq 0$, using Lemma 2.5 we have

$$det(A_5) = \frac{\begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} & a_{14} & a_{15} \\ a_{22} & a_{23} & a_{24} & a_{25} \\ a_{32} & a_{33} & a_{34} & a_{35} \\ a_{42} & a_{43} & a_{44} & a_{45} \end{vmatrix} \\ \\ \begin{vmatrix} a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} & a_{24} & a_{25} \\ a_{32} & a_{33} & a_{34} & a_{35} \\ a_{42} & a_{43} & a_{44} & a_{45} \\ a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}}. \tag{1}$$

Besides, we know that all $a_{22}, a_{23}, a_{24}, a_{32}, a_{33}, a_{34}, a_{42}, a_{43}, a_{44}$ are nonzero numbers. So, using Lemma 2.6 for all $4 \times 4$ within (1), we have

$$det(A_5) = \frac{\begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}}, \tag{2}$$

where $S_{ij}(\forall i, j = 1, 2)$ is equal to

$$\frac{\begin{vmatrix} \dfrac{\begin{vmatrix} \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix} & \begin{vmatrix} a_{i(j+1)} & a_{i(j+2)} \\ a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \end{vmatrix} \\ \begin{vmatrix} a_{(i+1)j} & a_{(i+1)(j+1)} \\ a_{(i+2)j} & a_{(i+2)(j+1)} \end{vmatrix} & \begin{vmatrix} a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \\ a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \end{vmatrix} \end{vmatrix}}{a_{(i+1)(j+1)}} & \dfrac{\begin{vmatrix} \begin{vmatrix} a_{i(j+1)} & a_{i(j+2)} \\ a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \end{vmatrix} & \begin{vmatrix} a_{i(j+2)} & a_{i(j+3)} \\ a_{(i+1)(j+2)} & a_{(i+1)(j+3)} \end{vmatrix} \\ \begin{vmatrix} a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \\ a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \end{vmatrix} & \begin{vmatrix} a_{(i+1)(j+2)} & a_{(i+1)(j+3)} \\ a_{(i+2)(j+2)} & a_{(i+2)(j+3)} \end{vmatrix} \end{vmatrix}}{a_{(i+1)(j+2)}} \\ \dfrac{\begin{vmatrix} \begin{vmatrix} a_{(i+1)j} & a_{(i+1)(j+1)} \\ a_{(i+2)j} & a_{(i+2)(j+1)} \end{vmatrix} & \begin{vmatrix} a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \\ a_{(i+2)(j+1)} & a_{(2+1)(j+2)} \end{vmatrix} \\ \begin{vmatrix} a_{(i+2)j} & a_{(i+2)(j+1)} \\ a_{(i+3)j} & a_{(i+3)(j+1)} \end{vmatrix} & \begin{vmatrix} a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \\ a_{(i+3)(j+1)} & a_{(i+3)(j+2)} \end{vmatrix} \end{vmatrix}}{a_{(i+2)(j+1)}} & \dfrac{\begin{vmatrix} \begin{vmatrix} a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \\ a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \end{vmatrix} & \begin{vmatrix} a_{(i+1)(j+2)} & a_{(i+1)(j+3)} \\ a_{(i+2)(j+2)} & a_{(2+1)(j+3)} \end{vmatrix} \\ \begin{vmatrix} a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \\ a_{(i+3)(j+1)} & a_{(i+3)(j+2)} \end{vmatrix} & \begin{vmatrix} a_{(i+2)(j+2)} & a_{(i+2)(j+3)} \\ a_{(i+3)(j+2)} & a_{(i+3)(j+3)} \end{vmatrix} \end{vmatrix}}{a_{(i+2)(j+2)}} \end{vmatrix}}{\begin{vmatrix} a_{(i+1)(j+1)} & a_{(i+1)(j+2)} \\ a_{(i+2)(j+1)} & a_{(i+2)(j+2)} \end{vmatrix}}.$$

Using Definition 2.3, we obtain

$$\frac{\begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}}$$

$$
= \left( \frac{\begin{bmatrix}
\begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{vmatrix} \\ \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{vmatrix} & \begin{vmatrix} a_{14} & a_{15} \\ a_{24} & a_{25} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} & \begin{vmatrix} a_{24} & a_{25} \\ a_{34} & a_{35} \end{vmatrix} \end{vmatrix} \\
\begin{vmatrix} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \\ \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix} & \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} \\ \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} & \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} & \begin{vmatrix} a_{24} & a_{25} \\ a_{34} & a_{35} \end{vmatrix} \\ \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} & \begin{vmatrix} a_{34} & a_{35} \\ a_{44} & a_{45} \end{vmatrix} \end{vmatrix} \\
\begin{vmatrix} \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix} & \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} \\ \begin{vmatrix} a_{41} & a_{42} \\ a_{51} & a_{52} \end{vmatrix} & \begin{vmatrix} a_{42} & a_{43} \\ a_{52} & a_{53} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} & \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \\ \begin{vmatrix} a_{42} & a_{43} \\ a_{52} & a_{53} \end{vmatrix} & \begin{vmatrix} a_{43} & a_{44} \\ a_{53} & a_{54} \end{vmatrix} \end{vmatrix} &
\begin{vmatrix} \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} & \begin{vmatrix} a_{34} & a_{35} \\ a_{44} & a_{45} \end{vmatrix} \\ \begin{vmatrix} a_{43} & a_{44} \\ a_{53} & a_{54} \end{vmatrix} & \begin{vmatrix} a_{44} & a_{45} \\ a_{54} & a_{55} \end{vmatrix} \end{vmatrix}
\end{bmatrix}_{3\times 3}}{\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}_{3\times 3}} \right)^{*} . \tag{3}
$$

Clearly using Definition 2.4, the top part of fraction (3) is equal to $DC(DC(A_5))$, consequently (2) and (3) give

$$
det(A_5) = \left( \frac{DC(DC(A_5))}{\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}_{3\times 3}} \right)^{*} .
$$

The theorem is proved. □

Notice that in the Theorem 3.1, if the matrix $\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}$ is not doubly nonsingular or if some elements of it are zero, then by adding a multiple of one row to another row, or a multiple of one column to another column of the main matrix $A_5$, these problems can be resolved (since the determinant of the main matrix does not change).

**Example 3.2.** Let $A_5 = \begin{bmatrix} 3 & 5 & 1 & 0 & 4 \\ 2 & 1 & 6 & 3 & 2 \\ 4 & 3 & 2 & 2 & 5 \\ 1 & 6 & 1 & 3 & 4 \\ 7 & 5 & 4 & 4 & 3 \end{bmatrix}_{5\times 5}$. To calculate the $det(A_5)$, we first obtain $DC(DC(A_5))$ as follows:

$$\xrightarrow{DC(A_5)} \begin{bmatrix} -7 & 29 & 3 & -12 \\ 2 & -16 & 6 & 11 \\ 21 & -9 & 4 & -7 \\ -37 & 19 & -8 & -7 \end{bmatrix}_{4\times 4} \xrightarrow{DC(DC(A_5))} \begin{bmatrix} 54 & 222 & 105 \\ 318 & -10 & -86 \\ 66 & -4 & -84 \end{bmatrix}_{3\times 3}.$$

Now, by Theorem 3.1 we have

$$det(A_5) = \left( \frac{\begin{bmatrix} 54 & 222 & 105 \\ 318 & -10 & -86 \\ 66 & -4 & -84 \end{bmatrix}_{3\times 3}}{\begin{bmatrix} 1 & 6 & 3 \\ 3 & 2 & 2 \\ 6 & 1 & 3 \end{bmatrix}_{3\times 3}} \right)^{*} = \frac{\begin{vmatrix} \frac{\begin{vmatrix} \frac{54}{1} & \frac{222}{6} \\ \frac{318}{3} & \frac{-10}{2} \end{vmatrix}}{\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix}} & \frac{\begin{vmatrix} \frac{222}{6} & \frac{105}{3} \\ \frac{-10}{2} & \frac{-86}{2} \end{vmatrix}}{\begin{vmatrix} 6 & 3 \\ 2 & 2 \end{vmatrix}} \\ \frac{\begin{vmatrix} \frac{318}{3} & \frac{-10}{2} \\ \frac{66}{6} & \frac{-4}{1} \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 1 \end{vmatrix}} & \frac{\begin{vmatrix} \frac{-10}{2} & \frac{-86}{2} \\ \frac{-4}{1} & \frac{-84}{3} \end{vmatrix}}{\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}} \end{vmatrix}}{\begin{vmatrix} 1 & 6 & 3 \\ 3 & 2 & 2 \\ 6 & 1 & 3 \end{vmatrix}} = \frac{\begin{vmatrix} 262 & -236 \\ 41 & -8 \end{vmatrix}}{\begin{vmatrix} 1 & 6 & 3 \\ 3 & 2 & 2 \\ 6 & 1 & 3 \end{vmatrix}} = -1516.$$

## 4. Conclusion

We presented a new method to compute the determinant of a $5 \times 5$ matrix. In fact, this is a generalization of a simpler method for $4 \times 4$ matrices which was previously provided in [2]. It seems to be possible to generalize this method for matrices of order $n$. Of course, for more generalizations, more calculations are required.

## Competing Interests

The author do not have any competing interests in the manuscript.


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