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SUPER (a, d)-C_3-ANTIMAGICNESS OF A CORONA GRAPH

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ABSTRACT. A simple graph $G = (V(G), E(G))$ admits an H-covering if $\forall\ e \in E(G) \ \Rightarrow\ e \in E(H')$ for some $(H' \cong H) \subseteq G$. A graph G with H covering is an (a, d)-H-antimagic if for bijection $f : V \cup E \rightarrow \{1, 2, \ldots, |V(G)| + |E(G)|\}$, the sum of labels of all the edges and vertices belong to H' constitute an arithmetic progression $\{a, a+d, \ldots, a+(t-1)d\}$, where t is the number of subgraphs H'. For $f(V) = \{1, 2, 3, \ldots, |V(G)|\}$, the graph G is said to be *super* (a, d)-H-*antimagic* and for $d = 0$ it is called H-*supermagic*. In this paper, we investigate the existence of super (a, d)-C_3-antimagic labeling of a corona graph, for differences $d = 0, 1, \ldots, 5$.

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Key words and phrases: star graph S_n; corona graph; C_3-supermagic; super (a, d)-C_3-antimagic.

1. Introduction

Let G be a simple graph with vertex set V and edge set E. An *edge-covering* of finite and simple graph G is a family of subgraphs $H_1, H_2, \ldots, H_t$ such that each edge of $E(G)$ belongs to at least one of the subgraphs H_i, $i = 1, 2, \ldots, t$. In this case we say that G admits an $(H_1, H_2, \ldots, H_t)$-*(edge) covering*. If every subgraph H_i is isomorphic to a given graph H, then the graph G admits an H-*covering*. A graph G admitting an H-covering is called (a, d)-H-*antimagic* if there exists a total labeling $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such

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that for each subgraph H' of G isomorphic to H, the H'-weights,

$$wt_f(H') = \sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e),$$

constitute an arithmetic progression $a, a+d, a+2d, \ldots, a+(t-1)d$, where $a > 0$ and $d \geq 0$ are two integers and t is the number of all subgraphs of G isomorphic to H.
The (super) H-magic graph was first introduced by Gutiérrez and Lladó in [1]. The (a,d)-H-antimagic labeling was introduced by Inayah *et al.* [2].
In [3] Bača *et al.* investigated the super tree-antimagic total labelings of disjoint union of graphs. Bača *et al.* [4] showed the constructions for H-antimagicness of Cartesian product of graphs. In [5], authors proved the C_n-antimagicness of Fan graph for several difference depending on the length of the cycle. In [6, 7, 8], Umar *et al.* proved the existence of super $(a,1)$-Tree-antimagicness of Sun graphs, super (a,d)-C_n-antimagicness of Windmill graphs for several differences and super (a,d)-C_4-antimagicness of Book graph and their disjoint union.
In this paper, we study the existence of super (a,d)-C_3-antimagic labeling of a special type of a corona graph.

2. Super Cycle-antimagic labeling of Corona graph

The *join* of two graphs H_1 and H_2, denoted by $H_1 + H_2$, is the graph where $V(H_1) \cap V(H_2) = \emptyset$ and each vertex of H_1 is adjacent to all vertices of H_2 [9]. When $H_1 = K_1$, this is the corona graph $K_1 \odot H_2$. In this paper, we consider a special type of a corona graph.
Let K_1 be a complete graph and S_n be a star on $n+1$ vertices. We consider the corona graph $G = K_1 \odot S_n$, where

$$V(G) := \{v_1, v_2, x_1, x_2, \ldots, x_n\}$$

and

$$E(G) := \{v_1v_2, v_1x_1, v_1x_2, \ldots, v_1x_n, v_2x_1, v_2x_2, \ldots, v_2x_n\}$$

The corona graph G is covered by the cycles $C_3^{(i)}$, $1 \leq i \leq n$ and the $C_3^{(i)}$-weights under a labeling h is:

$$\begin{aligned} wt_h(C_3^{(i)}) &= \sum_{v\in V(C_3^{(i)})} h(v) + \sum_{e\in E(C_3^{(i)})} h(e) \\ &= h(v_1) + h(v_2) + h(x_i) + h(v_1v_2) + h(v_1x_i) + h(v_2x_i) \qquad (1) \end{aligned}$$

2.1. C_3-Supermagic labeling.

Theorem 2.1. *Let $G := K_1 \odot S_n$ be a corona graph of K_1 and S_n and $n \geq 2$ be an integer then the graph G admits a C_3-supermagic labeing.*

Proof. When $n \equiv 0 \pmod 2$
The labeling h_0 is defined as:

$$\begin{aligned} h_0(v_1) &= 1, \\ h_0(v_2) &= \frac{n}{2} + 2, \\ h_0(v_1 v_2) &= 3n + 3, \\ h_0(v_1 x_i) &= 3n + 3 - i. \end{aligned}$$

$$h_0(x_i) = \begin{cases} \frac{n}{2} + 2 - i & \text{if } i = 1, 2, \dots, \frac{n}{2} \\ \frac{3n+6}{2} - i & \text{if } i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n \end{cases}$$

$$h_0(v_2 x_i) = \begin{cases} n + 2(1 + i) & \text{if } i = 1, 2, \dots, \frac{n}{2} \\ 2i + 1 & \text{if } i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n \end{cases}$$

Clearly, the vertices assume least possible integers $\{1, 2, \dots, n + 2\}$ under the labeling h_0 and edges receive the labels $\{n + 3, n + 4, \dots, 3n + 3\}$. Therefore h_0 is a super total labeling.
Using equation (1)

$$\begin{aligned} wt_{h_0}(C_3^{(i)}) &= \sum_{v \in V(C_3^{(i)})} h(v) + \sum_{e \in E(C_3^{(i)})} h(e) \\ &= \left(\frac{7n}{2} + 6\right) + \left(\frac{9n}{2} + 7\right) \\ &= 8n + 13. \end{aligned} \tag{2}$$

When $n \equiv 1 \pmod 2$
The labeling h_0 is defined as:

$$\begin{aligned} h_0(v_i) &= i, \\ h_0(v_1 v_2) &= n + 3, \\ h_0(x_i) &= n + 3 - i. \end{aligned}$$

For $i \equiv 0 \pmod 2$

$$h_0(v_j x_i) = \begin{cases} n + 3 + \frac{i}{2} & \text{if } j = 1 \\ \frac{5n+7+i}{2} & \text{if } j = 2 \end{cases}$$

For $i \equiv 1 \pmod 2$

$$h_0(v_j x_i) = \begin{cases} \frac{3(n+2)+i}{2} & \text{if } j = 1 \\ \frac{4n+7+i}{2} & \text{if } j = 2 \end{cases}$$

Clearly, the vertices assume least possible integers $\{1, 2, \dots, n + 2\}$ under the labeling h_0 and edges receive the labels $\{n + 3, n + 4, \dots, 3n + 3\}$. Therefore h_0

is a super total labeling.
Using equation (1)

$$\begin{aligned} wt_{h_0}(C_3^{(i)}) &= \sum_{v\in V(C_3^{(i)})} h(v) + \sum_{e\in E(C_3^{(i)})} h(e) \\ &= (2n+9-i) + \left(\frac{7n+13}{2} + i\right) \\ &= \frac{11n+31}{2}. \end{aligned} \tag{3}$$

Equations (2, 3) shows $wt_{h_0}(C_3^{(i)})$ is independent of i. Hence the corona graph G admits a C_3-supermagic labeling. This completes the proof. □

2.2. Super (a,d)-C_3-antimagic labeling.

Theorem 2.2. *Let $G := K_1 \odot S_n$ be a corona graph of K_1 and S_n and $n \geq 2$ be an integer then the graph G admits a super $(a,1)$-C_3-antimagic labeing.*

Proof. The labeling h_1 is defined as:

$$\begin{aligned} h_1(v_i) &= i, \\ h_1(v_1v_2) &= n+3, \\ h_1(v_2x_i) &= 2n+3+i. \end{aligned}$$

$$h_1(x_i) = \begin{cases} \frac{i+1}{2} + 2 & \text{if } i \equiv 1 \pmod 2 \\ \lceil \frac{n}{2} \rceil + 2 + \frac{i}{2} & \text{if } i \equiv 0 \pmod 2 \end{cases}$$

$$h_1(v_1x_i) = \begin{cases} \frac{4n+7-i}{2} & \text{if } i \equiv 1 \pmod 2 \\ \lceil \frac{n-1}{2} \rceil + n + 4 - \frac{i}{2} & \text{if } i \equiv 0 \pmod 2 \end{cases}$$

Clearly, the vertices assume least possible integers $\{1, 2, \ldots, n+2\}$ under the labeling h_1 and edges receive labels $\{n+3, n+4, \ldots, 3n+3\}$. Therefore h_1 is a super total labeling.
Using equation (1)

$$\begin{aligned} wt_{h_1}(C_3^{(i)}) &= \sum_{v\in V(C_3^{(i)})} h(v) + \sum_{e\in E(C_3^{(i)})} h(e) \\ &= 3(n+3) + i + (2n+6) \\ &= 5(n+3) + i. \end{aligned} \tag{4}$$

Equation (4) shows $wt_{h_0}(C_3^{(i)})$ constitute an arithmetic progression with $a = 5(n+3)+1$ and $d = 1$. Hence the corona graph G admits a super $(a,1)$-C_3-antimagic labeling. This completes the proof. □

Theorem 2.3. *Let $G := K_1 \odot S_n$ be a corona graph of K_1 and S_n and $n \geq 2$ be an integer then the graph G admits a super (a,d)-C_3-antimagic labeing for $d = 3, 5$.*

Proof. The labeling h_d is defined as:

$$h_d(v_i) = i,$$
$$h_d(v_1v_2) = n + 3,$$
$$h_d(x_i) = 2 + i.$$

$$h_3(v_jx_i) = \begin{cases} 2n + 3 + i & \text{if } j = 1 \\ n + 3 + i & \text{if } j = 2 \end{cases}$$

$$h_5(v_jx_i) = \begin{cases} n + 2 + 2i & \text{if } j = 1 \\ n + 3 + 2i & \text{if } j = 2 \end{cases}$$

Clearly, the vertices assume least possible integers $\{1, 2, \ldots, n + 2\}$ under the labeling h_d and edges receive labels $\{n+3, n+4, \ldots, 3n+3\}$. Therefore h_d is a super total labeling.

Using equation (1)

$$\begin{aligned} wt_{h_3}(C_3^{(i)}) &= \sum_{v \in V(C_3^{(i)})} h(v) + \sum_{e \in E(C_3^{(i)})} h(e) \\ &= (n + 8 + i) + (3n + 6 + 2i) \\ &= 2(2n + 7) + 3i. \end{aligned} \tag{5}$$

Equation (5) shows $wt_{h_3}(C_3^{(i)})$ constitute an arithmetic progression with $a = 2(2n + 7) + 3$ and $d = 3$. Hence the corona graph G admits a super $(a, 3)$-C_3-antimagic labeling.

Now, for case $d = 5$, Using equation (1)

$$\begin{aligned} wt_{h_5}(C_3^{(i)}) &= \sum_{v \in V(C_3^{(i)})} h(v) + \sum_{e \in E(C_3^{(i)})} h(e) \\ &= (n + 8 + i) + (2n + 5 + 4i) \\ &= 3n + 13 + 5i. \end{aligned} \tag{6}$$

Equation (6) shows $wt_{h_3}(C_3^{(i)})$ constitute an arithmetic progression with $a = 3(n+6)$ and $d = 5$. Hence the corona graph G admits a super $(a, 5)$-C_3-antimagic labeling. This completes the proof. □

Theorem 2.4. *Let $G := K_1 \odot S_n$ be a corona graph of K_1 and S_n and $n \geq 2$ be an integer then the graph G admits a super (a, d)-C_3-antimagic labeing for $d = 2, 4$.*

Proof. The labeling h_d is defined as:

$$h_d(v_i) = i$$

$$h_d(x_i) = \begin{cases} n + 3 - i & \text{if } d = 2 \\ 2 + i & \text{if } d = 4 \end{cases}$$

The edges are labeled as:
When $n \equiv 0 \pmod 2$

$$h_d(v_1v_2) = 5\left(\frac{n}{2}\right) + 3$$

$$h_d(v_1x_i) = \begin{cases} n+2+i & \text{if } i = 1, 2, \ldots, \frac{n}{2}+1 \\ \frac{n}{2}+1+2i & \text{if } i = \frac{n}{2}+2, \frac{n}{2}+3, ..., n \end{cases}$$

$$h_d(v_2x_i) = \begin{cases} \frac{3n}{2}+2(1+i) & \text{if } i = 1, 2, \ldots, \frac{n}{2}+1 \\ 2n+3+i & \text{if } i = \frac{n}{2}+2, \frac{n}{2}+3, ..., n \end{cases}$$

Clearly, the vertices assume least possible integers $\{1, 2, \ldots, n+2\}$ under the labeling h_d and edges receive labels $\{n+3, n+4, \ldots, 3n+3\}$. Therefore h_d is a super total labeling.
Using equation (1)

$$\begin{aligned} wt_{h_2}(C_3^{(i)}) &= \sum_{v \in V(C_3^{(i)})} h(v) + \sum_{e \in E(C_3^{(i)})} h(e) \\ &= \left(\frac{7n}{2} + 9 - i\right) + \left(\frac{5n}{2} + 4 + 3i\right) \\ &= 6n + 13 + 2i. \end{aligned} \tag{7}$$

Equation (7) shows $wt_{h_2}(C_3^{(i)})$ constitute an arithmetic progression with $a = 6n+15$ and $d = 2$. Hence the corona graph G admits a super $(a, 2)$-C_3-antimagic labeling. Using equation (1)

$$\begin{aligned} wt_{h_4}(C_3^{(i)}) &= \sum_{v \in V(C_3^{(i)})} h(v) + \sum_{e \in E(C_3^{(i)})} h(e) \\ &= (\frac{5n}{2} + 8 + i) + (\frac{5n}{2} + 4 + 3i) \\ &= 5n + 12 + 4i. \end{aligned} \tag{8}$$

Equation (8) shows $wt_{h_4}(C_3^{(i)})$ constitute an arithmetic progression with $a = 5n+16$ and $d = 4$. Hence the corona graph G admits a super $(a, 4)$-C_3-antimagic labeling.
When $n \equiv 1 \pmod 2$

$$h_d(v_1v_2) = 3n + 3$$

$$h_d(v_1x_i) = \begin{cases} n+2+i & \text{if } i = 1, 2, \ldots, \frac{n+1}{2} \\ \frac{n+1}{2}+1+2i & \text{if } i = \frac{n+1}{2}+1, \frac{n+1}{2}+2, ..., n \end{cases}$$

$$h_d(v_2x_i) = \begin{cases} \frac{n+1}{2}+n+1+2i & \text{if } i = 1, 2, \ldots, \frac{n+1}{2} \\ 2(n+1)+i & \text{if } i = \frac{n+1}{2}+1, \frac{n+1}{2}+2, ..., n \end{cases}$$

Clearly, the vertices assume least possible integers $\{1,2,\ldots,n+2\}$ under the labeling h_d and edges receive labels $\{n+3,n+4,\ldots,3n+3\}$. Therefore h_d is a super total labeling.
Using equation (1)

$$\begin{aligned} wt_{h_2}(C_3^{(i)}) &= \sum_{v\in V(C_3^{(i)})} h(v) + \sum_{e\in E(C_3^{(i)})} h(e) \\ &= (4n+9-i) + \left(\frac{5n+7}{2} + 3i\right) \\ &= \frac{13n+25}{2} + 2i. \end{aligned} \tag{9}$$

Equation (9) shows $wt_{h_2}(C_3^{(i)})$ constitute an arithmetic progression with $a = \frac{13n+29}{2}$ and $d=2$. Hence the corona graph G admits a super $(a,2)$-C_3-antimagic labeling.
Using equation (1)

$$\begin{aligned} wt_{h_4}(C_3^{(i)}) &= \sum_{v\in V(C_3^{(i)})} h(v) + \sum_{e\in E(C_3^{(i)})} h(e) \\ &= (3n+8+i) + \left(\frac{5n+7}{2} + 3i\right) \\ &= \frac{11n+23}{2} + 4i. \end{aligned} \tag{10}$$

Equation (10) shows $wt_{h_4}(C_3^{(i)})$ constitute an arithmetic progression with $a = \frac{11n+31}{2}$ and $d=4$. Hence the corona graph G admits a super $(a,4)$-C_3-antimagic labeling. This completes the proof. □

Competing Interests

The author(s) do not have any competing interests in the manuscript.

References

1. Gutiérrez, A. & Lladó, A. (2005). Magic coverings. *J. Combin. Math. Combin. Comput.* 55, 43–56.
2. Inayah, N., Salman, A. N. M., & Simanjuntak, R. (2009). On (a,d)-H-antimagic coverings of graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing, 71,* 273–281.
3. Bača, M., Kimăkovă, Z., Feňovčíková, A. Semaničová, & Umar, M. A. (2015). Tree-antimagicness of disconnected graphs. *Mathematical Problems in Engineering, Volume 2015,* Article ID 504251, 4 pages http://dx.doi.org/10.1155/2015/504251
4. Bača, M., Feňovčíková, A. Semaničová, Umar, M.A., & D. Welyyanti. (2018). On H-antimagicness of Cartesian product of graphs. *Turkish Journal of Mathematics, 42*(1), 339-348. http://dx.doi.org/10.3906/mat-1704-86.
5. Ovais, A., Umar, M.A., Bača, M., & Feňovčíková, A.Semaničová. (2017). Fans are cycle-antimagic, *The Australasian Journal of Combinatorics*, 6(1), 94–105.

6. Umar, M. A., Hussain, M., Ali, B. R., & Numan M. (2018). Super $(a, 1)$-Tree-antimagicness of Sun Graphs. *International Journal of Soft Computing and Engineering*, 7(6), 4 pages.
7. Umar, M.A., & Hussain M. (2018). Super (a, d)-C_n-antimagicness of Windmill Graphs, *International Journal of Science & Engineering Development Research*, 3(2), 5 pages.
8. Umar, M. A., Javed, M. A., Hussain, M. & Ali, B. R. (2018). Super (a, d)-C_4-antimagicness of Book Graphs, *Open Journal of Mathematical Sciences*, 2(1), 115–121. http://dx.doi.org/10.30538/oms2018.0021.
9. Bača, M., & Youssef, Maged Z. (2015). On Harmonious Labeling of Corona Graphs, *Journal of Applied Mathematics, Volume 2014* Article ID 627248, 4 pages http://dx.doi.org/10.1155/2014/627248.

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