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3-total edge mean cordial labeling of some standard graphs

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Abstract: In this paper, we introduce new labeling and named it as k-total edge mean cordial (k-TEMC) labeling. We study certain classes of graphs namely path, double comb, ladder and fan in the context of 3-TEMC labeling.

Keywords: 3-TEMC labeling, path graph, double comb graph, ladder graph, fan graph.

MSC: Primary 05C78, Secondary 05C15.

1. Introduction and Preliminaries

We begin with finite, undirected, simple and connected graph $\mathcal{G} = (V_{\mathcal{G}}, E_{\mathcal{G}})$. The set $V_{\mathcal{G}}$ is called vertex set and the set $E_{\mathcal{G}}$ is called edge set of graph $\mathcal{G}$. Order of a graph is the number of vertices in $\mathcal{G}$ and size of a graph is the number of edges in $\mathcal{G}$. We follow the standard notations and terminology of graph theory as in [1]. Graph labeling were first introduced in the late $1960's$. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (or edges) then the labeling is called a *vertex labeling* (or an *edge labeling*). We have the following notations, in order to know cordial labeling α and its sorts .

1. The number of vertices labeled by x is $v_\alpha(x)$;
2. The number of edges labeled by x is $e_\alpha(x)$;
3. $v_\alpha(x,y) = v_\alpha(x) - v_\alpha(y)$;
4. $e_\alpha(x,y) = e_\alpha(x) - e_\alpha(y)$;
5. $s(x)$=$v_\alpha(x) + e_\alpha(x)$;
6. $\mathbb{Z}_k$ denotes the first k non negative integers, i.e $\mathbb{Z}_k = \{0, 1, \ldots, k-1\}$.

Cordial labeling was introduced by Cahit in [2]. Now we will define cordial labeling and its different types.

Definition 1. Let $\alpha : V_{\mathcal{G}} \to \mathbb{Z}_2$ be a mapping that induces $\alpha^* : E_{\mathcal{G}} \to \mathbb{Z}_2$ as $\alpha^*(uv) = |\alpha(u) - \alpha(v)|$ where $uv \in E_{\mathcal{G}}$. Then α is called *cordial labeling* if $|v_\alpha(1,0)| \leq 1$ and $|e_\alpha(1,0)| \leq 1$.

Definition 2. Let $\alpha : V_{\mathcal{G}} \to \mathbb{Z}_2$ be a mapping that induces $\alpha^* : E_{\mathcal{G}} \to \mathbb{Z}_2$ as $\alpha^*(uv) = \alpha(u)\alpha(v)$ where $uv \in E_{\mathcal{G}}$. Then α is called *product cordial labeling* if $|v_\alpha(1,0)| \leq 1$ and $|e_\alpha(1,0)| \leq 1$. For details see [3].

Definition 3. Let $\alpha : V_{\mathcal{G}} \to \mathbb{Z}_2$ be a mapping that induces $\alpha^* : E_{\mathcal{G}} \to \mathbb{Z}_2$ as $\alpha^*(uv) = \alpha(u)\alpha(v)$ where $uv \in E_{\mathcal{G}}$. Then α is called *total product cordial labeling* if $|s(0) - s(1)| \leq 1$. For details see [4,5].

Definition 4. Let $\alpha : V_{\mathcal{G}} \to \mathbb{Z}_k$, $2 \leq k \leq |E_{\mathcal{G}}|$ be a mapping that induces $\alpha^* : E_{\mathcal{G}} \to \mathbb{Z}_k$ as $\alpha^*(uv) = \alpha(u)\alpha(v)$ $(\text{mod} k)$ where $uv \in E_{\mathcal{G}}$. Then α is called a *k-total product cordial labeling* if $|s(a) - s(b)| \leq 1$ for all $a, b \in \mathbb{Z}_k$. For details see [6].

Definition 5. Let $\alpha : E_{\mathcal{G}} \to \mathbb{Z}_2$ be a mapping that induces $\alpha^* : V_{\mathcal{G}} \to \mathbb{Z}_2$ such that $\alpha^*(u) = \alpha(e_1)\alpha(e_2)\ldots\alpha(e_n)$ for edges $e_1, e_2, \ldots, e_n$ incident to u, then α is called *edge product cordial labeling* if $|v_\alpha(0,1)| \leq 1$ and $|e_\alpha(0,1)| \leq 1$. For details see [7,8].

Definition 6. Let $\alpha : E_G \rightarrow \mathbb{Z}_2$ be a mapping that induces $\alpha^* : V_G \rightarrow \mathbb{Z}_2$ such that $\alpha^*(u) = \alpha(e_1)\alpha(e_2)\ldots\alpha(e_n)$ for edges $e_1, e_2, \ldots, e_n$ that are incident to u, then α is called a *total edge product cordial labeling* if $|s(0) - s(1)| \leq 1$. For details see [9,10].

Definition 7. Let $\alpha : E_G \longrightarrow \mathbb{Z}_k$, $2 \leq k \leq |E_G|$ be a mapping that induces $\alpha^* : V_G \longrightarrow \mathbb{Z}_k$ such that $\alpha^*(u) = \alpha(e_1)\alpha(e_2)\ldots\alpha(e_n)(\text{mod } k)$ for edges $e_1, e_2, \ldots, e_n$ incident to u, then α is called *k-total edge product cordial labeling* if it satisfy $|s(a) - s(b)| \leq 1$ for all $a, b \in \mathbb{Z}_k$. For details see [11–19].

Motivated by the above definitions, we introduce new labeling named k-total edge mean cordial labeling which is defined as:

Definition 8. Let $\alpha : E_G \longrightarrow \mathbb{Z}_k$ be a mapping that induces $\alpha^* : V_G \longrightarrow \mathbb{Z}_k$ such that for each vertex u, $\alpha^*(u)$ $=\lceil \alpha(e_1) + \alpha(e_2)\ldots + \alpha(e_n)/n \rceil$, where $e_1, e_2, \ldots, e_n$ are incident with u then α is called *k-total edge mean cordial labeling* (k-TEMC) if $|s(i) - s(j)| \leq 1$ where $i, j \in \mathbb{Z}_k$. A graph with a k-total edge mean cordial labeling is called k-total edge mean cordial graph.

The rest of the paper is structured as follows: In Section 2, 3-TEMC labeling of path is discussed. Section 3 devoted to the study 3-TEMC labeling of double comb graph. In Section 4, we study ladder graph and its 3-TEMC labeling. In Section 5, 3-TEMC labeling of fan graph is discussed.

2. 3-TEMC labeling of path graph

A *path graph* P_n is a graph whose vertices can be listed in the order $u_1, u_2, u_3, \ldots, u_n$ such that the edges are $u_1u_2, u_2u_3, \ldots, u_{n-1}u_n$. Here $V_{P_n} = \{u_i : 1 \leq i \leq n\}$ and $E_{P_n} = \{e_i = u_iu_{i+1} : 1 \leq i \leq n-1\}$. (see Figure 1).

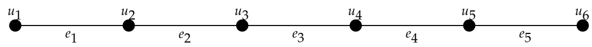

Figure 1. Path graph P_6

Next theorem tells us the 3-TEMC labeling of path graph.

Theorem 9. *Any path P_n for $n \geq 3$ admits 3-TEMC labeling.*

Proof. The following cases should be considered in order to prove that P_n is 3-TEMC.

Case 1 . If $n \equiv 0 \ (\text{mod } 3)$ then $n = 3q$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of P_3 (see Figure 2).

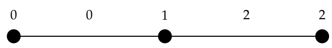

Figure 2. 3-TEMC labeling of P_3

If $q \geq 2$, then we define the function $\alpha : E(P_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q-1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 2q, & \text{if } i = 0, 1 \\ 2q-1, & \text{if } i = 2. \end{cases}$.

Case 2 . Let $n \equiv 1 \ (\text{mod } 3)$ then $n = 3q+1$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of P_4 (see Figure 3).

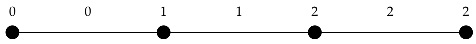

Figure 3. 3-TEMC labeling of P_4

If $q \geq 2$, then we define the function $\alpha : E(P_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 2q, & \text{if } i = 0,1 \\ 2q+1, & \text{if } i = 2. \end{cases}$.

Case 3 . If $n \equiv 2 \pmod 3$ then $n = 3q+2$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of P_5 (see Figure 4).

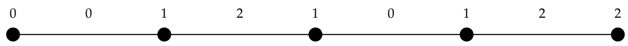

Figure 4. 3-TEMC labeling of P_5

If $q \geq 2$, then we define the function $\alpha : E(P_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+3 \leq i \leq 2q+1; \quad \text{and } \alpha(e_{q+2}) = 0, \alpha(e_{q+1}) = 1. \\ 2, & \text{if } 2q+2 \leq i \leq 3q+1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = 2q+1$, for all $i = 0, 1, 2$.

Hence P_n have 3-TEMC labeling for $n \geq 2$.

□

3. 3-TEMC labeling of ladder graph

The *ladder* graph L_n is defined as the cartesian product of P_n by K_2 where P_n is a path with n vertices and K_2 is a complete graph with two vertices. Here $V_{L_n} = \{u_i, v_i : 1 \leq i \leq n\}$ and $E_{L_n} = \{e_i = u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{f_i = u_i v_i, 1 \leq i \leq n\} \cup \{g_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$(see Figure 5).

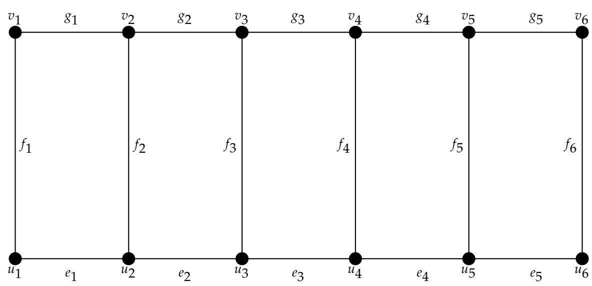

Figure 5. Ladder graph L_6

Next theorem tells us the 3-TEMC labeling of ladder graph.

Theorem 10. *Let L_n be a ladder graph, then L_n admits 3-TEMC labeling.*

Proof. The following cases should be considered in order to prove that L_n is 3-TEMC.

Case 1 . If $n \equiv 0 \pmod 3$ then $n = 3q$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of L_3 (see Figure 6).

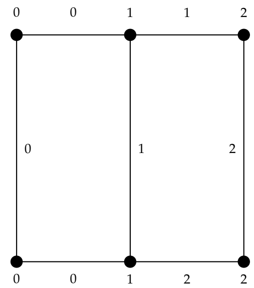

Figure 6. 3-TEMC labeling of L_3

If $q \geq 2$, then we define the function $\alpha : E(L_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q-1; \\ 2, & \text{if } 2q \leq i \leq 3q-1. \end{cases} \quad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases}$$

$$\alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q+1; \\ 2, & \text{if } 2q+2 \leq i \leq 3q-1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 5q, & \text{if } i = 0 \\ 5q-1, & \text{if } i = 1,2. \end{cases}$

Case 2 . If $n \equiv 1 \pmod 3$ then $n = 3q+1$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of L_4 (see Figure 7).

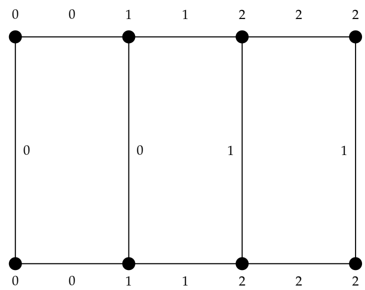

Figure 7. 3-TEMC labeling of L_4

If $q \geq 2$, then we define the function $\alpha : E(L_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases} \quad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+2; \\ 2, & \text{if } 2q+3 \leq i \leq 3q+1. \end{cases}$$

$$\alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = 5q+1$, for all $i = 0,1,2$.

Case 3 . If $n \equiv 2 \pmod 3$ then $n = 3q + 2$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of L_5 (see Figure 8).

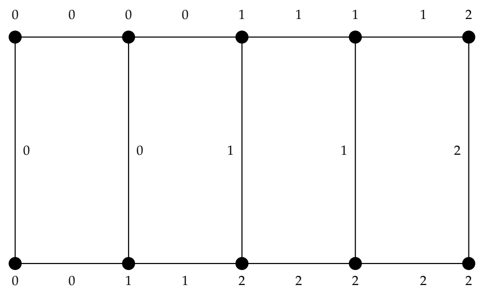

Figure 8. 3-TEMC labeling of L_5

If $q \geq 2$, then we define the function $\alpha : E(L_n) \to \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q+1. \end{cases} \qquad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+2; \\ 2, & \text{if } 2q+3 \leq i \leq 3q+2. \end{cases}$$

$$\alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+2; \\ 2, & \text{if } 2q+3 \leq i \leq 3q+1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 5q+3, & \text{if } i = 0, 1 \\ 5q+2, & \text{if } i = 2. \end{cases}$

Hence the ladder graph L_n have 3-TEMC labeling.

□

4. 3-TEMC labeling of double comb graph

Double comb graph DCO_n is a graph achieved by unification of two pendant edges u_iv_i and u_iw_i to every vertex u_i of a path graph. Here $V_{DCO_n} = V_{P_n} \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E_{DCO_n} = E_{P_n} \cup \{f_i = u_iv_i, g_i = u_iw_i : 1 \leq i \leq n\}$ (see Figure 9).

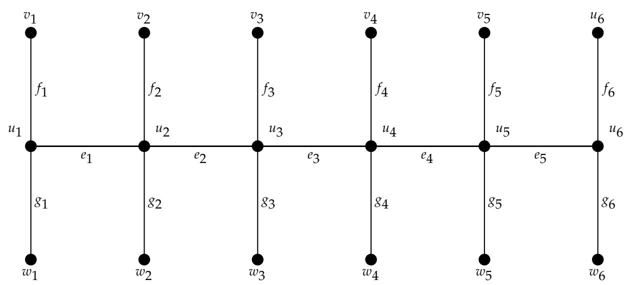

Figure 9. Double comb graph DCO_6

Next theorem tells us the 3-TEMC labeling of double comb graph.

Theorem 11. *Let DCO_n be a double comb graph, then DCO_n admits 3-TEMC labeling.*

Proof. The following cases should be considered in order to prove that DCO_n is 3-TEMC.

Case 1 . If $n \equiv 0 \pmod 3$ then $n = 3q$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of DCO_3 (see Figure 10).

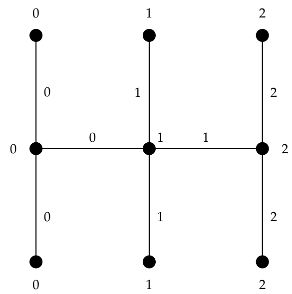

Figure 10. 3-TEMC labeling of DCO_3

If $q \geq 2$, then we define the function $\alpha : E(DCO_n) \to \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q-1. \end{cases} \quad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases} \quad \alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 6q, & \text{if } i = 0, 1 \\ 6q-1, & \text{if } i = 2. \end{cases}$

Case 2 . If $n \equiv 1 \pmod 3$ then $n = 3q+1$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of DCO_4 (see Figure 11).

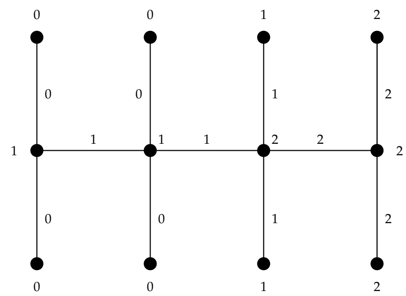

Figure 11. 3-TEMC labeling of DCO_4

If $q \geq 2$, then we define the function $\alpha : E(DCO_n) \to \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q-1; \\ 1, & \text{if } q \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases} \quad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+1; \\ 2, & \text{if } 2q+2 \leq i \leq 3q+1. \end{cases}$$

$$\alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+1; \\ 2, & \text{if } 2q+2 \leq i \leq 3q+1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 6q+2, & \text{if } i = 0, 1 \\ 6q+1, & \text{if } i = 2. \end{cases}$

Case 3 . If $n \equiv 2 \pmod 3$ then $n = 3q + 2$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of DCO_5 (see Figure 12).

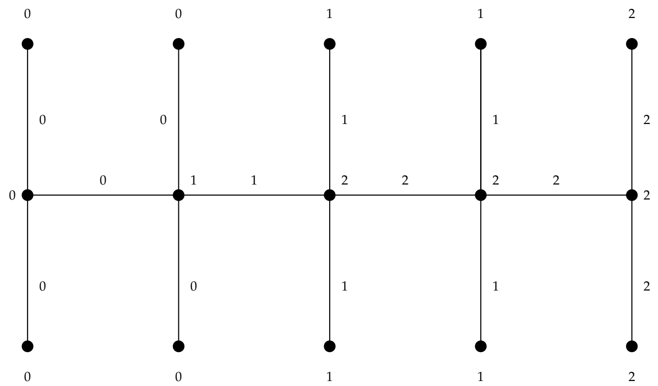

Figure 12. 3-TEMC labeling of DCO_5

If $q \geq 2$, then we define the function $\alpha : E(DCO_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q+1. \end{cases} \quad \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+2; \\ 2, & \text{if } 2q+3 \leq i \leq 3q+2. \end{cases}$$

$$\alpha(g_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q+1; \\ 1, & \text{if } q+2 \leq i \leq 2q+2; \\ 2, & \text{if } 2q+3 \leq i \leq 3q+2. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 6q+4, & \text{if } i = 0,1 \\ 6q+3, & \text{if } i = 2. \end{cases}$

Hence the double comb graph DCO_n have 3-TEMC labeling.

□

5. 3-TEMC labeling of fan graph

A *Fan graph* F_n is the graph obtained by taking n copies of the cycle graph C_3 with a vertex u in common. Here $V_{F_n} = \{uu_i : 1 \leq i \leq 2n\}$ and $E_{F_n} = \{e_i = uu_i : 1 \leq i \leq 2n\} \cup \{f_i = u_iu_{i+1} : 1 \leq i \leq n-1 \text{ and } i \text{ is odd}\}$ (see Figure 13).

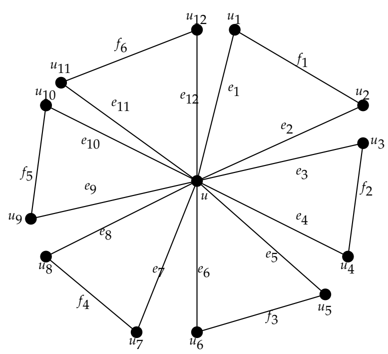

Figure 13. Fan graph F_6

Next theorem tells us the 3-TEMC labeling of fan graph.

Theorem 12. *Fan graph F_n for $n \geq 2$ admits 3-TEMC labeling.*

Proof. The following cases should be considered in order to prove that F_n is 3-TEMC.

Case 1 . Let $n \equiv 1(\text{mod } 3)$ then $n = 3q$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of F_3 (see Figure 14).

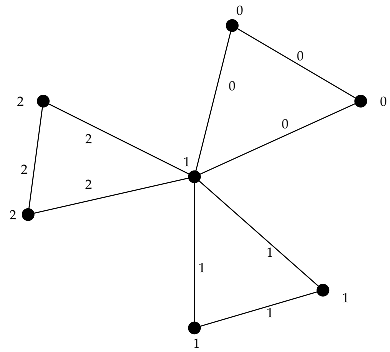

Figure 14. 3-TEMC labeling of F_3

If $q \geq 2$, then we define the function $\alpha : E(F_n) \to \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq 2q; \\ 1, & \text{if } 2q+1 \leq i \leq 4q; \\ 2, & \text{if } 4q+1 \leq i \leq 6q. \end{cases} \alpha(f_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq q; \\ 1, & \text{if } q+1 \leq i \leq 2q; \\ 2, & \text{if } 2q+1 \leq i \leq 3q. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 5q, & \text{if } i = 0, 2 \\ 5q+1, & \text{if } i = 1. \end{cases}$

Case 2 . Let $n \equiv 1 \ (\text{mod } 3)$ then $n = 3q + 1$, where $q \in \mathbb{Z}^+$. For $q = 1$, we have the following labeling of F_4 (see Figure 15).

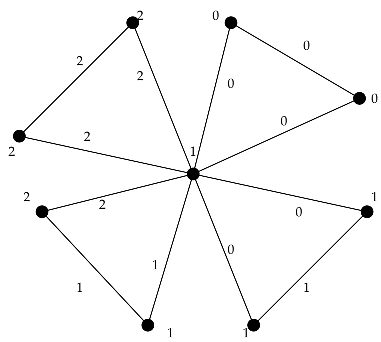

Figure 15. 3-TEMC labeling of F_4

If $q \geq 2$, then we define the function $\alpha : E(F_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq 2q+2; \\ 1 & \text{if } 2q+3 \leq i \leq 4q+1; \\ 2 & \text{if } 4q+2 \leq i \leq 6q+2. \end{cases} \alpha(f_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq q; \\ 1 & \text{if } q+1 \leq i \leq 2q+1; \\ 2 & \text{if } 2q+2 \leq i \leq 3q+1. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = 5q+2$, for all $i = 0, 1, 2$.

.

Case 3 . If $n \equiv 1 \pmod 3$ then $n = 3q+2$, where $q \in \mathbb{Z}^+$. we have the following labeling of F_2 (see Figure 16).

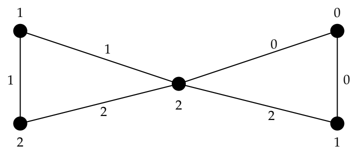

Figure 16. 3-TEMC labeling of F_2

If $q \geq 1$, then we define the function $\alpha : E(F_n) \rightarrow \mathbb{Z}_3$ as

$$\alpha(e_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq 2q+1; \\ 1 & \text{if } 2q+2 \leq i \leq 4q+3; \\ 2 & \text{if } 4q+4 \leq i \leq 6q+4. \end{cases} \alpha(f_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq q+1; \\ 1 & \text{if } q+2 \leq i \leq 2q+1; \\ 2 & \text{if } 2q+2 \leq i \leq 3q+2. \end{cases}$$

Hence α is 3-TEMC labeling because $s(i) = \begin{cases} 5q+3, & \text{if } i = 0 \\ 5q+4, & \text{if } i = 1, 2. \end{cases}$

Hence the fan graph F_n have 3-TEMC labeling for $n \geq 2$.

□

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