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Vector calculus and Maxwell's equations: logic errors in mathematics and electrodynamics

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Abstract: The critical analysis of the foundations of vector calculus and classical electrodynamics is proposed. Methodological basis of the analysis is the unity of formal logic and rational dialectics. The main results are the following statements: (1) a vector is a property of the motion and of the interaction of material objects, i.e., the concept of a vector is the concept of a physical property. Therefore, the concept of a vector is a general and abstract concept; (2) a vector is depicted in the form of an arrow (i.e., "straight-line segment with arrowhead") in a real (material) coordinate system. A vector drawn (depicted) in a coordinate system does not have the measure "meter". Therefore, a vector is a pseudo-geometric figure in a coordinate system. A vector is an imaginary (fictitious) geometric figure; (3) geometrical constructions containing vectors (as pseudo-geometric figures) and vector operations in a coordinate system are fictitious actions; (4) the scalar and vector products of vectors represent absurd because vectors (as abstract concepts, as fictional geometric figures that have different measures) cannot intersect at the material point of the coordinate system; (5) the concepts of gradient, divergence, and rotor as the basic concepts of vector analysis are a consequence of the main mathematical error in the foundations of differential and integral calculus. This error is that the definition of the derivative function contains the inadmissible operation: the division by zero; (6) Maxwell's equations the main content of classical electrodynamics are based on vector calculus. This is the first blunder in the foundations of electrodynamics. The second blunder is the methodological errors because Maxwell's equations contradict to the following points: (a) the dialectical definition of the concept of measure; (b) the formal-logical law of identity and the law of lack of contradiction. The logical contradiction is that the left and right sides of the equations do not have identical measures (i.e., the sides do not have identical qualitative determinacy). Thus, vector calculus and classical electrodynamics represent false theories.

Keywords: Dimensional analysis, Euclidean and projective geometries, vector, vector fields, Maxwell equations.

MSC: 00A05, 00A30, 00A35, 00A69.

1. Introduction

As is known, vector calculus represents an important part of the mathematical formalism of theoretical physics [1–10]. The physical theories using (containing) vector calculus look elegant and convincing theories [11–20]. But apparent elegance and convincingness of the standard theories created by such classics of sciences as Isaac Newton, Gottfried Leibniz, William Hamilton, Hermann Grassmann, Josiah Gibbs, Oliver Heaviside, James Maxwell, Heinrich Hertz *et al*. [11–20] does not signify the validity of the standard theories.

The apparent elegance and convincingness of standard theories is not a sign, feature, proof and criterion of truth of the theories. This statement is based on the fact that the classics of sciences did not find the correct criterion of truth of theories. They could not find the correct criterion of truth of theories because they could not find the correct methodological basis of science: the unity of formal logic and rational dialectics. In this point of view, vector calculus and Maxwell's theory of electromagnetic field should be called in question within the framework of the correct methodological basis. Also, forcible argument is that the foundations of mathematics and theoretical physics contain formal-logical and dialectical errors [21–24].

The purpose of this work is to propose the critical analysis of the foundations of vector calculus and of classical electrodynamics within the framework of the correct methodological basis: the unity of formal logic and rational dialectics. The dialectical analysis is based on the dialectical concept of measure: the measure of material object is the unity of qualitative and quantitative determinacy of the material object; the measure of physical quantity is the unity of qualitative and quantitative determinacy of the physical quantity. The formal-logical analysis is based on the law of identity and the law of lack of contradiction: the correct mathematical equation represents the relationship between the identical measures of the identical physical quantities. This signifies that the sides of the mathematical equation must have identical measures, i.e., both sides of the mathematical equation must belong to identical qualitative determinacy.

2. General statements

(a) In accordance with formal logic, the research of the Universe is possible if and only if the researcher divides mentally the Universe into the following aspects: the material and informational aspects. Thereupon the researcher must divide mentally the material aspect into the following aspects: physical aspect, geometric aspect, chemical aspect, biological aspect, etc. The next step in the research is that the researcher must choose one certain aspect for research: the physical aspect or the geometric aspect. These aspects have different essential features and, therefore, should be researched separately.

(b) Physics studies the physical measure of a material object. Physics operates with physical (abstract) concepts, i.e., the concepts of the physical properties of material objects or material phenomena. These properties are abstracted from material objects or material phenomena and represent an independent object of thought.

(c) Geometry studies the geometric (material) object and the geometric measure of the material object. Geometry operates with geometric (material) objects and geometric (abstract) concepts, i.e., the concepts of the geometric properties of material objects. These properties are abstracted from material objects and represent an independent object of thought.

(d) The correct methodological basis for research represents the unity of formal logic and rational dialectics. The correct methodological basis for research is the criterion of truth. The concept of measure as the unity of the qualitative and quantitative determinacy of the material object is a central concept in sciences.

(e) A mathematical (i.e., quantitative) description of properties is that properties are expressed with the help denominate (concrete) numbers (i.e., numbers that have names). Therefore, the mathematical relationship represents a quantitative relationship that belongs to a certain qualitative determinacy of a material object. In other words, a mathematical relationship as a unity of qualitative and quantitative determinacy expresses the measure of a material object. This signifies that both sides of the mathematical (quantitative) relationship must have identical qualitative determinacy.

(f) But there are classes of relationships and operations that cannot be expressed in mathematical form. For example, the equations of chemical reactions, the operation of adding vectors in physics, the projection operation in geometry. But all operations must be carried out within the framework of the correct methodological basis. In this point of view, the concepts "physical geometry" and "geometric physics" represent a methodological error.

(g) Material objects have a measure: the unity of qualitative and quantitative determinacy. The measure characterizes the material properties (length, area, volume, mass, etc.) of the object, the properties of motion and interaction (speed, acceleration, direction, directionality, force, etc.). Properties do not exist without material objects. The concept of property is an abstract concept.

3. Geometrical statements

(1) A geometric object is a material object that has the following measure (quantitative determinacy): extent (length). The extent (length) is measured in units of length ("meter"). The only qualitative determinacy (i.e., an essential sign, feature, property) of a geometric object is materiality. In other words, a geometric object is a material object that does not have physical properties. The concept "geometric object" is a general and concrete concept.

The concept of a geometric object includes (covers) material points, material lines, material figures, and material bodies. A material point represents a specific geometric object that has the measure "zero meter" (i.e., a material point has zero extent). Therefore, material points belonging to geometric objects are identical points. A geometric space is a set of states (i.e., positions) of geometric objects in a geometric (material) system of reference.

Geometric (i.e., material) objects exist in a geometric (material) system of reference. A geometric system of reference is a material geometric object is called a coordinate system and contains planes and measuring rulers (measuring devices).

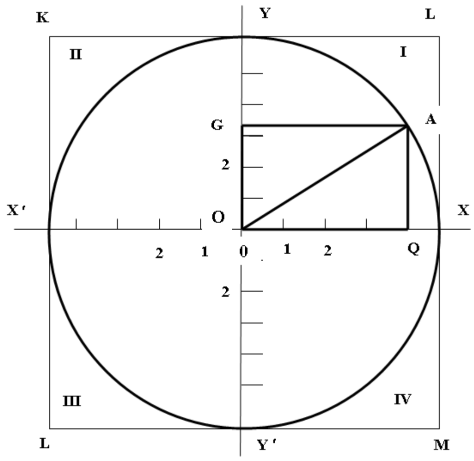

Figure 1. Cartesian coordinate system XOY as a material system. The circle, rectangle and triangle are the material geometrical figures in the coordinate system XOY.

(2) In accordance with practice, the ruler is a device for measuring the length (i.e., measure) of a material object. The result of measuring the length is a number with the dimension "meter". This number does not determine the direction (directivity) of the material object in the coordinate system. Therefore, set of rulers do not determine the direction (directivity) of a material object (for example, a rod, a rocket, a planet); the direction of a material object (for example, a rod, a rocket, a planet) does not characterize set of rulers.

In accordance with practice, an angle is a system of two material segments. The result of measuring the angle is a number with the dimension "degree". This number does not determine the direction (directivity) of the material segments. Therefore, set of angles do not determine the direction (directivity); direction (directivity) does not characterize set of angles.

(3) Cartesian (geometric) coordinate system $XOYZ$ is a set of connected material rulers OX, OY, OZ (on which numbers with dimension "meter" are indicated) and material planes XOY, XOZ, YOZ . An example of a Cartesian system is shown in Figure 1.

The description of the system XOY and of geometric operations in the system XOY is the following.

(a) The horizontal scale represents the rulers OX and OX' which are connected at the common point O . The vertical scale represents the rulers OY and OY' which are connected at the common point O. The common point O is the origin of coordinates. The number 0 on the scales is a reference point. The number 0 is a neutral number (i.e., a number without the signs "$+$" and "$-$"). Therefore, in accordance with formal logic, numbers $1, 2, 3, ...$ on the scales are also neutral numbers. All these numbers have the dimension (name) "meter". The existence of numbers on the scales is not a feature (criterion) of the direction of the rulers.

(b) The horizontal and vertical scales are on the material plane $KLMN$. The plane $KLMN$ is a flat surface of a solid body. The scales divide the plane $KLMN$ into the quadrants I, II, III, IV, i.e., $XOY, X'OY, X'OY'XOY'$, respectively.

(c) One of the important geometric operations in the Cartesian coordinate system is the operation of projection of a geometric object (material point, material line, material figure, and material body). As is known, the operation of projection is defined by the following way: the operation of projection of a geometric object onto a coordinate ruler (scale) is the operation of constructing an image (depiction) of a geometric object on the coordinate ruler (scale). In other words, the operation of projection is the operation of coincidence of the image (depiction) of a geometric object with a coordinate ruler (scale). The result of the operation represents the respective segment on the coordinate ruler (scale). It lead to the following formulation of the principle of the existence of a geometric object on a coordinate ruler (scale): a geometric object on a coordinate ruler (scale) exists as a straight line segment on the coordinate ruler (scale).

(d) The rectangle $OGAQ$ as a material figure is in the material circle that is on the plane $KLMN$. The segment $\overrightarrow{OA}$ is the radius of the circle and the diagonal of the rectangle $OGAQ$. The segments $\overrightarrow{OQ}$ and $\overrightarrow{OG}$ are the material projections of the rectangle $OGAQ$, the diagonal $\overrightarrow{OA}$, the radius $\overrightarrow{OA}$, and the triangles $\triangle OGA$

,$\triangle OQA$. The lengths of the segments $\overrightarrow{OQ}$ and $\overrightarrow{OG}$ represent the quantities $|\overrightarrow{OQ}|$ and $|\overrightarrow{OG}|$ which take on numerical values.

(e) The projections $\overrightarrow{OQ}$ and $\overrightarrow{OG}$ are (lie) on the rulers OX and OY, respectively. The coincidence of the projections $\overrightarrow{OQ}$,$\overrightarrow{OG}$ and the rulers OX, OY, signifies that projections $\overrightarrow{OQ}$, $\overrightarrow{OG}$ and the rulers OX, OY have identical measures (i.e., "meter"). The notations (designations) of the segments $\overrightarrow{OA}$ and $\overrightarrow{AO}$ are identical notations (designations) $\overrightarrow{OA}$ and $\overrightarrow{AO}$ because they designate the same segment. In other words, the notations (designations) and signify the existence of this segment.

(f) All points of the segment $\overrightarrow{OA}$ are identical points. The boundary (end) points O and A belong to the segment $\overrightarrow{OA}$. Therefore, the points O and A are identical points. If the boundary (end) points O and A would signify the "origin" and the "end" of the segment $\overrightarrow{OA}$, then the points O and A would be non-identical to all other points of the segment $\overrightarrow{OA}$. In this case, the points O and A would not belong to the set of points of the segment $\overrightarrow{OA}$. But this assertion would contradict to the condition of existence of the segment $\overrightarrow{OA}$ as an element of geometric figures.

There is no mathematical operation that turns the points O and A into the "origin" and "end" of the segment $\overrightarrow{OA}$.

(g) The projection of the material point A is the material point Q on the ruler OX. But the segment $\overrightarrow{OQ}$ is not the projection of the point A because the point A (which has the measure 0 "meter") cannot turn into the segment $\overrightarrow{OQ}$ (which has the measure "meter"). The projection of a point is a point which is characterized by a named number on the coordinate ruler. This number indicates the distance of the point A from the point O.

(h) The area of the rectangle $OGAQ$ has the measure "square meter" (m^2) and represents a property of the rectangle. This property is characterized by the abstract concept "square meter" and does not exist in the coordinate system XOY.

(i) The quantity of the angle $\angle AOQ$ is a property of the material angle $\angle AOQ$ and has the measure "degree". This property is characterized by an abstract concept and does not exist in the coordinate system XOY. The rotation of the radius $\overrightarrow{OA}$ of the circle leads to a change in the value of the quantity of the angle. But the direction of rotation does not exist in the coordinate system XOY because the direction of rotation is a property of motion.

(j) The relationships between the quantities of the angles and the lengths of the sides in the right-angled triangle $\triangle AOQ$ represent a property of the material right-angled triangle. This property does not exist in the coordinate system XOY . The correct relationships between the quantities of the angles and the lengths of the sides can be expressed in mathematical form only within the framework of the systems approach.

(k) The operations of construction and transformation (change) of geometric figures are not described by mathematics.

4. Examples

The operation of integration of triangles (i.e., $\triangle OGA + \triangle AQO = quadrilateral\ OGAQ$) is not a mathematical operation. This is the geometric operation of the integration of figures. The areas of the figures represent the quantities that take on numerical values (denominate numbers). Therefore, the expression $n \cdot S_{\triangle OGA}$ (where $n = 0, 1, 2, ...$ the quantity $S_{\triangle OGA}$ is the area of the triangle) is a correct mathematical expression. But the expressions $n \cdot \triangle OGA$ and $|n \cdot \triangle OGA|$ represents absurd.

The operation of projection the segment $\overrightarrow{OA}$ is not a mathematical operation. The operation of projection the segment $\overrightarrow{OA}$ is the geometric operation of construction of the segments $\overrightarrow{OQ}$ and $\overrightarrow{OG}$. The lengths of the segments $\overrightarrow{OQ}$ and $\overrightarrow{OG}$ represent the quantities $|\overrightarrow{OQ}|$ and $|\overrightarrow{OG}|$ which take on numerical values (denominate numbers). Therefore, the expressions $n \cdot |\overrightarrow{OQ}|$ and $n \cdot |\overrightarrow{OG}|$ are correct mathematical expressions. But expressions $n \cdot \overrightarrow{OQ}$ and $n \cdot \overrightarrow{OG}$ represent absurd.

Material arrow is a geometric (material) figure. The projection (i.e., image, depiction) of an arrow onto a material coordinate ruler is a rectilinear material segment on the coordinate ruler. But the modulus $| \rightarrow |$ of the arrow as a mathematical expression is absurd.

(l) A geometric object does not have physical properties (i.e., mass, force, strength, momentum, energy) in the system $XOYZ$. A material object exists (i.e., is depicted) in the coordinate system $XOYZ$ if and only if the object is a geometric object (i.e., if the measure of the object is "meter"). A non-geometric object does not exist (i.e., is not depicted) in the system $XOYZ$.

(m) Properties (for example, length, direction, directivity, volume, mass, density, energy, etc.) of a material object are characterized by abstract concepts and are mathematically expressed by dimensional numbers. But the properties of a material object do not exist (i.e., is not depicted) in the system $XOYZ$. Also, the properties of motion (for example, speed, acceleration, momentum, energy, etc.) of a material object and the properties of the interaction of material objects are characterized by abstract concepts and are mathematically expressed by dimensional numbers. But the properties of motion do not exist (i.e., are not depicted) in the system $XOYZ$.

(n) The energy of a material object is a property, a sign, a feature (measure) of the movement of a material object. Energy space is the set of permissible values of the energy of a material object. Force (strength) is a property, a sign, a feature (measure) of the interaction of material objects. Energy, force (strength) and other properties can be measured, but the results of measurement do not exist (i.e., are not depicted) in the system $XOYZ$.

(o) Time is an informational (non-physical) property of a clock, i.e., property of the clockwork and hands moving on the dial. The clock determines the time (having the dimension "second"); time characterizes the clock. Time is an information quantity created and used by man to order information about events and processes in the world. Therefore, time as the property of the clock does not exist (i.e., is not depicted) in the system $XOYZ$.

5. Definition of the concept of vector

(a) As is known, "vector represents a property of the motion and interaction of material objects, i.e., the concept of vector is a physical concept. A vector is characterized by two essential features: the length of the rectilinear segment (i.e., the denominate quantity) and the direction of the rectilinear segment" [2]. In this case, the concepts "segment" and "direction" are inseparably connected concepts and form a unity. Therefore, vector is a directed segment. A directed segment exists in the system $XOYZ$ if a directed segment represents a geometric (i.e., material) figure.

(b) The concept of direction expresses the property of directivity of the influence, the property of directivity of the interaction and movement of material objects. This property is not characterized by the concepts "segment" and "length of segment" because the property is not a material object. A vector is not a geometric (material) object and, therefore, does not exist in the system $XOYZ$ because the concept of vector is general and abstract concept. Consequently, a vector is an imaginary (fictitious) geometric figure that is depicted in the form of an arrow (i.e., "straight-line segment with arrowhead") in the real (material) coordinate system . If an arrow drawn (depicted) in the system has the measure "meter", then this arrow is a geometric figure.

But if the arrow drawn (depicted) in the system $XOYZ$ does not have the measure "meter", then this arrow is a pseudo-geometric figure. A vector belongs to the class of pseudo-geometric figures. The projections of the vector (as an imaginary geometric figure) on the coordinate rulers represent imaginary straight-line segments (without an arrowhead) on the coordinate rulers. This signifies that geometrical constructions using a vector (as a pseudo-geometric figure) and vector operations in the system $XOYZ$ are fictitious actions.

Thus, the correct definition of the concept of vector is the following: a vector is not a material object; a vector is an abstract concept: the concept of a physical property; a vector is a pseudo-geometric figure: a vector is a depiction (image, picture) of an imaginary directed segment (i.e., a depiction, image, picture of an imaginary arrow).

6. Critical analysis of the foundations of vector algebra

(a) As is known, "the resolution (decomposition) of any vector $\vec{A}$ into components in the system $XOYZ$ (i.e., the resolution (decomposition) of the vector endwise the axes (axes expansion) of coordinates has the following form: $\vec{A} = x\vec{i} + y\vec{j} + z\vec{k}$ (where x, y, z are Cartesian coordinates of the vector $\vec{A}; \vec{i}, \vec{j}, \vec{k}$ are the unit vectors of coordinate axes)" [2].

But this resolution (decomposition) of the vector is an erroneous and inadmissible operation because: firstly,x, y, z are Cartesian coordinates of the point, i.e., x, y, z are distances of the points from the point O on the coordinate rulers. The projection of a point is a point on the ruler. A point has no length.

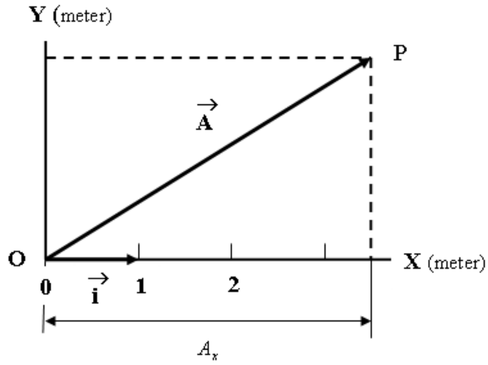

Figure 2. Standard images (depictions) of the vectors $\vec{A}$ and $\vec{i}$ in the forms of an arrow. If the arrow $\vec{A}$ drawn (depicted) in the system $XOYZ$ has the measure "meter", then the arrow is a geometric figure. But if the arrow $\vec{A}$ drawn (depicted) in the system $XOYZ$ does not have the measure "meter", then the arrow is a pseudo-geometric figure. A_x is the "projection length" of the vector $\vec{A}$.

Consequently,$x\vec{i} = y\vec{j} = z\vec{k} = absurdity$; secondly, a vector as a physical property (i.e., as an abstract concept) is characterized by a denominate number (i.e., a number having the dimension of a physical quantity) and, therefore, a vector does not represent the segment of the geometric line in the system $XOYZ$; thirdly, the material point O is not the origin of the vector because a vector is an abstract concept; fourthly, the modulus of the vector (i.e., quantity $|\vec{A}|$) as the modulus of the arrow $| \rightarrow |$ is a meaningless mathematical expression; fifthly, the modulus of the vector $\vec{A}$ (i.e., the quantity $|\vec{A}|$) and the projections of the vector do not have the measure "meter" and, therefore, are not on the coordinate rulers OX, OY, OZ and in the system $XOYZ$; sixthly, if the modulus of the unit vectors $\vec{i}, \vec{j}, \vec{k}$ are dimensionless numbers, then the unit vectors $\vec{i}, \vec{j}, \vec{k}$ cannot be (lie) on the coordinate rulers; seventhly, if the unit vectors $\vec{i}, \vec{j}, \vec{k}$ are (lie) on the coordinate rulers OX, OY, OZ and the modulus of the unit vectors have the dimensions "meter", then the following contradiction arises: the rulers OX, OY, OZ (i.e., geometric objects) acquire (get) physical properties, i.e., parts of the rulers turn into physical objects that cannot exist in the system $XOYZ$. In other words, the coincidence of physical objects (i.e., the unit vectors $\vec{i}, \vec{j}, \vec{k}$) and geometric objects (i.e., the rulers OX, OY, OZ) is the formal-logical error which represents a violation of the law of identity and the law of lack of contradiction. This coincidence would mean the identity of the qualitative determinacy of the unit vectors and of the rulers; eighthly, the vector $\vec{A}$, the unit vectors $\vec{i}, \vec{j}, \vec{k}$ and the rulers cannot have the common point because the vector , the unit vectors and the rulers do not have identical measures; eighthly, the expressions $x\vec{i}, y\vec{j}, z\vec{k}$ represent absurd because they are products of the numbers x, y, z and the pseudo-geometric figures, respectively; ninthly, the unit vectors $\vec{i}, \vec{j}, \vec{k}$ as the pseudo-geometric figures cannot exist on coordinate rulers. The unit vectors $\vec{i}, \vec{j}, \vec{k}$ could exist on the coordinate rulers only as unit segments (without arrowheads) with the measure "meter".

(b) As is known [2], a field is a part of space at each point $P(x, y, z)$ of which a certain scalar or vector physical quantity is given. A vector field is a part of space at each point $P(x, y, z)$ of which a certain vector quantity $\vec{A}(P)$ is given. If the origin of the vector $\vec{A}(P)$ is at the point O of the coordinate system $XOYZ$, then the projections of the vector $\vec{A}(P)$ on the coordinate axis are A_x, A_y, A_z (Figure 2).

Therefore, the decomposition of the vector $\vec{A}(P)$ into components in the system $XOYZ$ has the following form:

$$\vec{A}(P) = A_x(x, y, z)\vec{i} + A_y(x, y, z)\vec{j} + A_z(x, y, z)\vec{k} \gg .$$

This expression represents mathematical definition of the vector $\vec{A}(P)$ in the coordinate system $XOYZ$.

(c) But a physical field is not a geometric object in the system $XOYZ$. Consequently, a physical field does not exist in the system $XOYZ$. And the concept of "field point" does not have a geometric sense. This fact signifies that the point $P(x, y, z)$ represents a material point (in particular, a device) in the system $XOYZ$. If the material point $P(x, y, z)$ interacts with a physical field, then the properties (characteristics) of the field can be measured at this point.

(d) The projections (i.e., pseudo-geometric segments) $\bar{A}_x, \bar{A}_y, \bar{A}_z$ are not vectors. This fact signifies the following: firstly, any drawn vector (as a drawn arrow) represents a geometric figure; secondly, the operation of projecting the vector $\vec{A}(P)$ (as a pseudo-geometric figure) leads to the destruction of the vector $\vec{A}(P)$ (as a pseudo-geometric figure). In other words, the operation of projecting the vector $\vec{A}(P)$ violates the connection between the concepts "segment" and "direction". This fact is the proof that the vector $\vec{A}(P)$ is a figure.

(e) Coincidence of the pseudo-geometric segments $\bar{A}_x, \bar{A}_y, \bar{A}_z$ with the respective (corresponding) unit vectors and rulers is an inadmissible operation because the pseudo-geometric segments $\bar{A}_x, \bar{A}_y, \bar{A}_z$ unit vectors and rulers have different (non-identical) measures (qualitative determinacy).

(f) As is known, the formation of the directivity property of the pseudo-geometric segments $\bar{A}_x, \bar{A}_y, \bar{A}_z$ is expressed by the following postulate: $A_x\vec{i}, A_y\vec{j}, A_z\vec{k}$ where A_x, A_y, A_z are the lengths of the pseudo-geometric segments. But the multiplication of the quantities A_x, A_y, A_z by the respective (corresponding) unit vectors would only lead to a change in the unit vectors and would not lead to the formation (appearance) of $\vec{A}_x, \vec{A}_y, \vec{A}_z$ or $\overrightarrow{A_x i}, \overrightarrow{A_y j}, \overrightarrow{A_z k}$. If the vectors $\vec{A}_x, \vec{A}_y, \vec{A}_z$ existed, then this would mean that the vectors $\vec{A}_x, \vec{A}_y, \vec{A}_z$ are not projections (depictions, images) of the vector $\vec{A}$ on the coordinate rulers.

Moreover, the multiplication of quantities A_x, A_y, A_z by the respective (corresponding) unit vectors represents absurd because this operation is the multiplication of pseudo-geometric figures by numbers.

Thus, the decomposition of the vector $\vec{A}(P)$ into components in the system $XOYZ$ is an inadmissible geometric operation.

(g) As is known, "if the vectors have the common origin (i.e., intersect at the material point), then the scalar and vector products of vectors are defined by the following expressions, respectively:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos(\vec{A}, \vec{B}),$$

where $\cos(\vec{A}, \vec{B})$ is the cosine of the angle between the vectors (if the vectors have the common origin O at the point, i.e., if the vectors intersect at the material point O);

$$\vec{A} \times \vec{B} = \vec{C}$$

where $\vec{C}$ is the vector whose modulus is the area $|\vec{A}||\vec{B}| \sin(\vec{A}, \vec{B})$ of the parallelogram $\sin(\vec{A}, \vec{B})$; is the sine of the angle between the vectors (if the vectors have the common origin at the point O, i.e., if the vectors intersect at the material point O)"[2].

(h) In the case of the scalar product, the intersection of vectors at the point O would signify that the quantities $|\vec{A}|$ and $|\vec{B}|$ have identical dimensions. Also, in the case of the vector product, the intersection of vectors at the point O would signify that the quantities $|\vec{A}|$,$|\vec{B}|$ and $|\vec{C}|$ and $|\vec{B}|$ have identical dimensions. But the dimensions of the quantities $|\vec{A}|$ and $|\vec{B}|$ are different.

In addition, the dimension of the quantity $|\vec{C}|$ is not identical to the dimensions of the quantities $|\vec{A}|$ and $|\vec{B}|$. This means that the origin of the vector $\vec{C}$ is not in the common material point O, i.e., the segments that have the measures $|\vec{A}|$,$|\vec{B}|$ and $|\vec{C}|$ do not intersect at the common material point O in cases of scalar and vector products.

In addition, the expressions $|\vec{A}| \cos(\vec{A}, \vec{B})$ and $|\vec{B}| \cos(\vec{A}, \vec{B})$ represent the projections of the vectors $\vec{A}$ and $\vec{B}$, respectively. But the projection operation is impossible because the dimensions of the quantities $|\vec{A}|$ and $|\vec{B}|$ are different.

Moreover, the relationships between the quantities of the angle and the lengths of the sides (i.e., the sine and cosine of the angle) are valid only in the case of a material right-angled triangle. But the concepts $\cos(\vec{A}, \vec{B})$ and $\sin(\vec{A}, \vec{B})$ lose the geometric sense in the case of abstract concepts $|\vec{A}|$ and $|\vec{B}|$. In the point of view of formal logic, there is no logical relation (logical connection) between abstract concepts $|\vec{A}|$,$|\vec{B}|$ and the concrete concepts "material point O in the material system $XOYZ$ ", "material segments in the material system $XOYZ$".

Consequently, the formulas for scalar and vector products are absolutely incorrect. Thus, vector algebra represents an absolutely incorrect theory.

7. Critical analysis of the foundations of vector analysis

As is known [2], the concepts of gradient, divergence and rotor are the basic concepts of vector analysis.

(a) The gradient of the scalar function $u(P)$ is defined by the following expression:

$$grad\ u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k},$$

$$i.e.,\ \ grad\ u = \nabla u,$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

where ∇ is the nabla-vector (i.e., the Hamilton operator);

(b) Divergence of the vector field,

$$\vec{A}(P) = A_x(x,y,z)\vec{i} + A_y(x,y,z)\vec{j} + A_z(x,y,z)\vec{k},$$

is defined by the following expression:

$$div\ \vec{A}(P) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z},$$

$$i.e.,\ \ div\ \vec{A}(P) = \nabla \cdot \vec{A}(P)$$

where the values of the partial derivatives are taken at the point P.

(c) Rotor of the vector field is defined by the following expression:

$$rot\ \vec{A}(P) = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}.$$

(d) But the definitions of gradient, divergence and rotor represent errors because: firstly, these definitions are based on vector algebra; secondly, these definitions are based on differential and integral calculus which represents an erroneous theory. (As was shown in the works [25], differential calculus is based on the following inadmissible operation: division by zero). Therefore, differential equations should be replaced by algebraic equations (this was A. Einstein's dream).

8. Critical comments on Maxwell's electrodynamics

As is known, the formalisms of vector calculus and differential calculus are used in the construction of the theory of electromagnetic field. A correct analysis of the theory can be performed only on the basis of the dialectical concept of measure within the framework of formal logic. Measure is the unity of the qualitative and quantitative determinacy of an object (material object or object of thought). Application of the concept of measure to a mathematical (quantitative) equation signifies that both sides of the equation must belong to the identical qualitative determinacy of the object.

(a) As is known [11–20], the electromagnetic field is described by the following Maxwell's phenomenological equations in differential form

$$rot\ \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c} + \frac{\partial D}{\partial t}, \tag{1}$$

$$rot\ \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$div\ \vec{B} = 0, \tag{3}$$

$$div\ D = 4\pi\rho, \tag{4}$$

where D is the vector of the electric field strength depending on spatial coordinates and time t (the dimension is $MLT^{-3}I^{-1}$);ρ is the vector of the magnetic flux density (magnetic inductance vector) depending on spatial coordinates and time t (the dimension is $MT^{-2}I^{-1}$); ρ is the electric charge density in space (dimension is ITL^{-3}); $\vec{j}$ is the vector of the electric current density in space (the dimension is IL^{-2}); $\vec{D}$ is the vector of the electric displacement (the dimension is ITL^{-2}); $\vec{H}$ is the vector of the magnetic field strength in a material (the dimension is IL^{-1}).

Maxwell's equations determine the basic characteristics of the electromagnetic field $(\vec{E}, \vec{B}, \vec{D}, \vec{H})$ at each point in space at any point of time, if the sources of the field $\vec{j}$ and ρ as functions of coordinates and time are known.

Maxwell's equations do not represent a complete (closed) system of equations. Therefore, Maxwell's equations are supplemented by the following equations of state (equations of connection of vectors):

$$\vec{D} = \vec{D}(\vec{E}), \vec{B} = \vec{B}(\vec{H}), \vec{j} = \vec{j}(\vec{H}).$$

The equations of state describe the electromagnetic properties of a concrete medium. For example, in the case of vacuum:$\vec{D} \equiv \vec{E}$ and $\vec{B} \equiv \vec{H}$. In the case of isotropic mediums:$\vec{D} = \varepsilon\vec{E}, \vec{B} = \mu\vec{H}, \vec{j} = \sigma\vec{E}\vec{j}_{(extraneous)}$ where $\varepsilon(x, y, z)$ is the dielectric constant of the medium, $\mu(x, y, z)$ is the magnetic permeability of the medium, $\sigma(x, y, z)$ is the specific electrical conductivity of the medium, and $\vec{j}_{(extraneous)}$ is the density of external currents. The macroscopic characteristics ε, μ, σ of the medium should be found experimentally.

(b) Equation (1) expresses the following logical relations between the physical quantities:

$$(\text{the measure of the physical quantity } |\vec{H}|) = (\text{the measure of the physical quantity } |\vec{j}|),$$

$$(\text{the measure of the physical quantity } |\vec{H}|) = (\text{the measure of the physical quantity } |\vec{D}|),$$

$$(\text{the measure of the physical quantity } |\vec{j}|) = (\text{the measure of the physical quantity } |\vec{D}|).$$

But these relations contradict to the formal-logical law of identity and the law of lack of contradiction, respectively:

$$(\text{the measure of the physical quantity } |\vec{H}|) = (\text{the measure of the physical quantity } |\vec{H}|),$$

$$(\text{the measure of the physical quantity } |\vec{j}|) = (\text{the measure of the physical quantity } |\vec{j}|),$$

$$(\text{the measure of the physical quantity } |\vec{D}|) = (\text{the measure of the physical quantity } |\vec{D}|),$$

$$(\text{the measure of the physical quantity } |\vec{H}|) \neq (\text{the measure of the physical quantity } |\vec{j}|),$$

$$(\text{the measure of the physical quantity } |\vec{H}|) \neq (\text{the measure of the physical quantity } |\vec{D}|),$$

$$(\text{the measure of the physical quantity } |\vec{j}|) \neq (\text{the measure of the physical quantity } |\vec{D}|).$$

Consequently, Equation (1) represents a logical error.

(c) Equation (2) expresses the following logical relations between the physical quantities:

$$(\text{the measure of the physical quantity } |\vec{E}|) = (\text{the measure of the physical quantity } |\vec{B}|).$$

But this relation contradicts to the formal-logical law of identity and the law of lack of contradiction, respectively:

$$(\text{the measure of the physical quantity } |\vec{E}|) = (\text{the measure of the physical quantity } |\vec{E}|),$$

$$(\text{the measure of the physical quantity } |\vec{B}|) = (\text{the measure of the physical quantity } |\vec{B}|),$$

$$(\text{the measure of the physical quantity } |\vec{E}|) \neq (\text{the measure of the physical quantity } |\vec{B}|).$$

Consequently, Equation (2) represents a logical error.

(d) Equation (4) expresses the following logical relations between the physical quantities:

$$(\text{the measure of the physical quantity } |\vec{D}|) = (\text{the measure of the physical quantity}\rho).$$

But this relation contradicts to the formal-logical law of identity and the law of lack of contradiction, respectively:

$$(\text{the measure of the physical quantity } |\vec{D}|) = (\text{the measure of the physical quantity } |\vec{D}|),$$

$$(\text{the measure of the physical quantity } |\vec{\rho}|) = (\text{the measure of the physical quantity } |\vec{\rho}|),$$

$$(\text{the measure of the physical quantity } |\vec{D}|) \neq (\text{the measure of the physical quantity } |\vec{\rho}|).$$

Consequently, Equation (4) represents a logical error.

(e) The equations of state (equations of connection) $\vec{D} = \vec{D}(\vec{E}), \vec{B} = \vec{B}(\vec{H}), \vec{j} = \vec{j}(\vec{H})$ contradict to the formal logical law of identity and the law of lack of contradiction, respectively:

$$(\text{the measure of the physical quantity } |\vec{D}|) = (\text{the measure of the physical quantity } |\vec{D}|),$$

$$(\text{the measure of the physical quantity } |\vec{E}|) = (\text{the measure of the physical quantity } |\vec{E}|),$$

$$(\text{the measure of the physical quantity } |\vec{D}|) \neq (\text{the measure of the physical quantity } |\vec{E}|),$$

$$(\text{the measure of the physical quantity } |\vec{B}|) = (\text{the measure of the physical quantity } |\vec{B}|),$$

$$(\text{the measure of the physical quantity } |\vec{H}|) = (\text{the measure of the physical quantity } |\vec{H}|),$$

$$(\text{the measure of the physical quantity } |\vec{B}|) \neq (\text{the measure of the physical quantity } |\vec{H}|),$$

$$(\text{the measure of the physical quantity } |\vec{j}|) = (\text{the measure of the physical quantity } |\vec{j}|),$$

$$(\text{the measure of the physical quantity } |\vec{E}|) = (\text{the measure of the physical quantity } |\vec{E}|),$$

$$(\text{the measure of the physical quantity } |\vec{j}|) \neq (\text{the measure of the physical quantity } |\vec{E}|).$$

Therefore, the equations of state (equations of connection) represent a logical error.

(f) Thus, Maxwell's equations represent absurd.

9. Discussion

(a) As is known, the classics of mathematics and theoretical physics were outstanding scientists. But this fact is not proof of the truth of their works. Because the outstanding scientists could not find and formulate a general criterion for the truth of theories. Well-known philosophers and logicians could not find, formulate and propose the correct methodological basis of science.

The correct methodological basis represents a method of rational thinking. Rational thinking makes it possible for researchers to separate truth from falsehood in the process of inductive cognition of the world. Because science is an inductive cognition of the world. Therefore, modern science needs a revision of generally accepted theories within the framework of rational thinking.

(b) "Rational thinking" (Werner Heisenberg) is thinking within the framework of the correct methodological basis of science: the unity of formal logic and rational dialectics. Formal logic is the science of the laws of correct thinking. The laws of formal logic were formulated by Aristotle. Rational dialectics represents the materialistic dialectic corrected on the basis of formal logic. Therefore, the unity of formal logic and rational dialectics is a criterion of truth.

(c) At present, mathematics and theoretical physics contain set (great number!) of theorems and theories that contradict to the laws of formal logic and rational dialectics [21–24]. Removal of errors from the foundations of mathematics and theoretical physics leads to the abolition (deletion) of many standard theorems and theories (for example, the theory of relativity). This fact signifies a crisis in theoretical physics. In effect (if one thinks), this means that "science for science oneself" (A. Einstein) exists today. Below are important examples.

(d) The idea of mechanical motion besoted Isaac Newton $(1643 - 1727)$. Newton entered (introduced) the concept of movement (change) into the mathematical expression of the function $y = f(x)$ by means of the increment Δx of the argument x. But he could not understand the essence of the limiting process. He obtained the characteristics (properties) of movement (i.e., change) of the function y under lack of movement (i.e., change) of the argument x (i.e., under $\Delta x = 0$). Therefore, the definition of the derivative function $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ contains the following contradiction: $\Delta x \neq 0, \Delta x = 0$ are both of one expression. It represents the logical error [25]. In 1684, the canon of differential calculus was created by logician G. Leibniz. But Leibniz could not find, understand, and detect Newton's logical error. Thus, the First Absurdity in the form of differential and integral calculus entered into mathematics and physics.

(e) Josiah Gibbs $(1839 - 1903)$ and Oliver Heaviside $(1850 - 1925)$ completed the creation of the Second Absurdity: vector calculus. The physical idea of vector besoted J. Gibbs and O. Heaviside. But J. Gibbs and O. Heaviside did not find an adequate mathematical formulation of the physical idea of vector. The heavy way from the physical idea to the mathematical formulation (description, presentation) of the physical idea turned out to be a false path. The explanation is the following.

Firstly, any material object (for example, a physical field) has properties (i.e., essential features). But properties (i.e., essential features) do not represent a material object and do not exist without a material object. In other words, a property as a material object does not exist. A property exists only as an object of thought. Therefore, the concept of property as an independent object of thought is an abstract concept.

Secondly, the concept of a vector is an abstract concept. An abstract concept cannot exist in the form of a material object in a geometric coordinate system. An arrow drawn in a geometric (material) coordinate system is a geometric figure (i.e., a material object). But the vector as an abstract concept is not identical to the material arrow drawn in the coordinate system. Therefore, the vector drawn in the coordinate system is a fictional (imaginary) geometric figure.

Thirdly, projection of a vector (as a fictional geometric figure) onto a coordinate ruler represents a construction of image (depiction) of a vector on a coordinate ruler. Therefore, the projection (i.e., image, depiction) of the vector is a fictitious (imaginary) geometric segment.

Fourthly, a vector as a physical property of a material object has a certain physical measure. Therefore, the vectors having different measures cannot intersect at a material point of the coordinate system. In addition, vectors cannot be coincided with the ruler of the coordinate system.

Consequently, scientists did not understand that a correct mathematical description (representation) of a vector does not exist.

(f) Maxwell's equations represent the Third Absurdity. Maxwell's equations formulated by Oliver Heaviside and Heinrich Hertz do not represent the correct macroscopic (phenomenological) theory of the electromagnetic field. The explanation is the following.

Firstly, Maxwell's equations contradict to the dialectical concept of measure: the measure of physical quantity is the unity of qualitative and quantitative determinacy of the physical quantity; mathematical equation represents relationship between the identical measures of the identical physical quantities. The contradiction is that each of Maxwell's equations contains the physical quantities which have nonidentity measures.

Secondly, as the critical analysis of the foundations of theoretical physics shows, any phenomenological theory contains methodological errors if it is not formulated within the framework of the systems approach. (The systems approach includes the mathematical formalism of proportions. The formalism of proportions does not contradict to the concept of measure). This signifies that all standard phenomenological theories are essentially false theories because they are not formulated on the basis of proportions.

Thirdly, even scientific lie contains part of scientific truth. This part of scientific truth is manifested in working material devices. Inventors of working material devices are not experts in theories. Inventors are creative people. They think rationally, practically, and independently. Therefore, working material devices represent practical proof (evidence, demonstration) of the truth of these material devices. This fact signifies that logical relation between the concepts "theoretical truth" and "practical truth" is the partial coincidence relation. In this connection, hazard (danger) of advanced (leading) and uncontrolled development of technology arises: for instance, the known theoretical relationship $E = mc^2$ contradicts to formal-logical laws; but this relationship gave rise to atomic bomb.

Thus, these examples show that errors in mathematics and theoretical physics are not casual, incidental errors. The errors represent logical corollary of inductive (uncontrolled) development of sciences.

Therefore, the article, "On the modern crisis of theoretical physics" (1922) by A. Einstein, is an actual statement of the situation in science so far.

10. Conclusion

Thus, the critical analysis of the foundations of vector calculus and of classical electrodynamics carried out within the framework of the correct methodological basis, leads to the following main statements:

(1) A vector represents a property of the motion and of the interaction of material objects, i.e., the concept of a vector is the concept of a physical property. Therefore, the concept of a vector is a general and abstract concept.
(2) A vector is an imaginary (fictitious) geometric figure that is depicted in the form of an arrow (i.e., "straight-line segment with arrowhead") in the real (material) coordinate system $XOYZ$. The vector drawn in the system $XOYZ$ does not have the measure "meter". Therefore, a vector is a pseudo-geometric figure in the coordinate system. The mathematical definition of a vector does not exist.
(3) The projections of the vector (as an imaginary geometric figure) onto the coordinate rulers represent imaginary straight-line segments (without arrowhead) on the coordinate rulers. Therefore, geometrical constructions containing vectors (as pseudo-geometric figures) and vector operations in the system $XOYZ$ are fictitious actions.
(4) The coincidence of vector projections with the respective unit vectors and coordinate rulers is an inadmissible operation because projections, unit vectors and coordinate rulers have non-identical measures. Vectors with non-identical measures cannot intersect at the material point of the coordinate system.
(5) The decomposition of the vector into components in the coordinate system represents an inadmissible operation because the multiplication of the projection of the vector (i.e., the numerical value of the physical quantity) by the unit vector (as a fictitious geometric figure) is absurd.
(6) The scalar and vector products of vectors is absurd because vectors (as abstract concepts, as fictional geometric figures that have different measures) cannot intersect at the material point of the coordinate system.
(7) The concepts of gradient, divergence and rotor as the basic concepts of vector analysis represent a mathematical error. The error is that the mathematical definitions of the gradient, divergence and rotor are based on differential and integral calculus. The starting point and basis of differential calculus is the concept of the derivative function. But the definition of the derivative function contains a gross error: division by zero.
(8) Maxwell's equations $-$ the main content of classical electrodynamics $-$ are based on vector calculus. This is the first blunder in the foundations of electrodynamics. The second blunder is the methodological errors because Maxwell's equations contradict to the following points: (a) the dialectical definition of the concept of measure; (b) the formal-logical law of identity and the law of lack of contradiction. The logical contradiction is that the left and right sides of the equations do not have identical measures (i.e., the sides do not have identical qualitative determinacy).

Thus, vector calculus and classical electrodynamics represent false theories.

Conflicts of Interest: "The author declares no conflict of interest."

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