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On 3-total edge product cordial labeling of tadpole, book and flower graphs

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Abstract: In this paper, we will determine the 3-total edge product cordial (3-TEPC) labeling. We study certain classes of graphs namely tadpole, book and flower graphs in the context of 3-TEPC labeling.

Keywords: 3-total edge product cordial labeling, book graphs, flower graphs.

MSC: 05C78.

1. Introduction

We define with simple, finite and undirected graph $G = (E_G, V_G)$, where E_G and V_G is vertex set and edge set respectively. Cordinalities of there sets are called the size and order of G. We follow the standard notations and terminology of graph theory as in [1]. A graph labeling is an assignment to vertices or edges or both subject to certain conditions. If the domain is $V_G \cup E_G$ then we called the labeling is total labeling.

We have the following notations, in order to know cordial labeling h and its sorts.

1. $v_h(j)$ is the number of vertices labeled by j;
2. $e_h(j)$ is the number of edges labeled by j;
3. $v_h(i,j) = v_h(i) - v_h(j)$;
4. $e_h(i,j) = e_h(i) - e_h(j)$ and
5. the sum of all vertices and edges labeled by j is $sum(j)$ i.e. $sum(j) = v_h(j) + e_h(j)$.

Cahit [2] gave first concept of cordial labeling as a weaker version of graceful labeling. A vertex labeling $h : V_G \to \{0,1\}$ that induce an edge labeling $h^* : E_G \to \{0,1\}$ defined by $h^*(uv) = |h(u) - h(v)|$, for all $u, v \epsilon E_G$ if $|v_h(1) - v_h(0)| \leq 1$ and $|e_h(1) - e_h(0)| \leq 1$. The concept of *product cordial labeling* was introduced by Sundaram *et al.* in 2014. For details see [3]. In 2006 and 2012, Sundaram *et al.* develop the concept of *total product cordial labeling* and *k-total product cordial labeling*. For details see [4,5]. Vaidya and Barasara gave the concept of *edge product cordial labeling* and *total edge product cordial labeling*. For details see [6,7].

Let k be an integer, $2 \leq k \leq |E_G|$ an edge labeling $h : E_G \longrightarrow \{0, 1, \ldots, k-1\}$, with induced vertex labeling $h^* : V_G \longrightarrow \{0, 1, \ldots, k-1\}$ such that $h^*(u) = h(e_1)h(e_2)\ldots h(e_n)(\text{mod } k)$, where edges $e_1, e_2, \ldots, e_n$ are the edge incident to u, then h is called *k-total edge product cordial labeling* if $|sum(i) - sum(j)| \leq 1$ for $i, j \in \{0, 1, \ldots, k-1\}$.

In 2015, Azaizeh *et al.* [8] was introduced the basic concept of *k-total edge product cordial (k-TEPC) labeling*. Recently Azaizeh *et al.* investigated the 3-TEPC labeling for more families of graphs namely, path, circle, fan, double fan, triangular snake graph (see example [9,10]). Ahmad *et al.* [11] discussed 3-TEPC labeling of gear, web and helm graph. Ali *et al.* [12] investigated the 3-TEPC labeling for families of convex polytopes namely, double antiprism A_m, S_m and T_m. In 2018, Ahmad *et al.* [13] had discussed the 3-TEPC labeling for hexagonal grid. Recently, Ali *et al.* [14] investigated the 4-total edge product cordial labeling of some standard graphs. For more details see references [15–18].

Now we will define tadpole graph $(T_{p,q})$, book graph (B_q) and flower graph (Fl_q) graph.

Definition 1. The tadpole graph is obtained by connecting a cycle graph C_p (of order p) to a path graph P_q (of order q) with a bridge and is denoted by $T_{p,q}$ (see Figure 1).

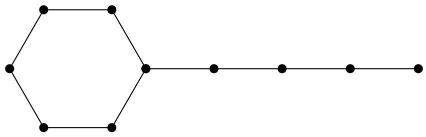

Figure 1. Tadpole graph $T_{6,4}$.

Definition 2. Book graph B_q is obtained by the cartesian product $S_{q+1} \times P_2$, where S_{q+1} is the star graph of order $q+1$ and P_2 is the path graph of order 2 (see Figure 2).

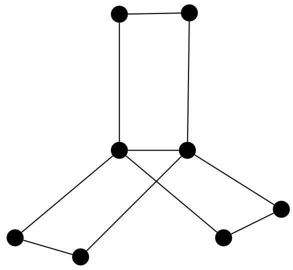

Figure 2. Book graph B_3

Definition 3. The flower graph Fl_q is the graph attain from a helm graph H_q of order q by joining each pendent vertex to the apex of the helm (see Figure 3).

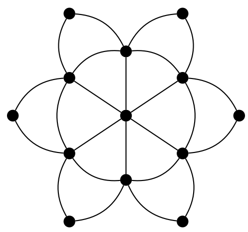

Figure 3. Flower graph Fl_6

2. Main results

In this section, we will discuss 3-total edge product cordial (3-TEPC) labeling of tadpole, book and flower graphs.

Theorem 1. *The tadpole graph $T_{p,q}$ has 3-TEPC labeling.*

Proof. Let $V_G = \{u_j, 1 \leq j \leq p\} \cup \{v_j, 1 \leq j \leq q\}$ and $E_G = \{u_j u_{j+1}, 1 \leq j \leq p-1\} \cup \{v_j v_{j+1}, 1 \leq j \leq q-1\} \cup \{u_p u_1\} \cup \{v_q u_p\}$ as shown in Figure 1. Now we see the following three cases:

Case 1 Let $p+q \equiv 0(\mod 3)$ which implies $p+q = 3t$, for $t \geq 3$. So for the given case, we need to discuss the following three subcases:

Case 1.1 If $p = 0(\mod 3)$ and $q = 0(\mod 3)$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_j u_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p}{3} - 1; \\ 2, & \text{if } \frac{p}{3} \leq j \leq p-1; \end{cases} \text{ and } h(u_p u_1) = 1.$$

$$h(v_j v_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q}{3}; \\ 2, & \text{if } \frac{q}{3} + 1 \leq j \leq q-1. \end{cases}, h(v_q u_p) = 2.$$

Case 1.2 If $p > q$ but $p \neq 0(\mod 3)$ and $q \neq 0(\mod 3)$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-2; \\ 2, & \text{if } t-1 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } j = 1; \\ 2, & \text{if } 2 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

Case 1.3 If $p < q$ but $p \neq 0(\mod 3)$ and $q \neq 0(\mod 3)$. We define $h : E_G \rightarrow \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } j = 1; \\ 2, & \text{if } 2 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-2; \\ 2, & \text{if } t-1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

So we obtain $sum(0) = sum(1) = sum(2) = 2t$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling as discussed in Figure 4.

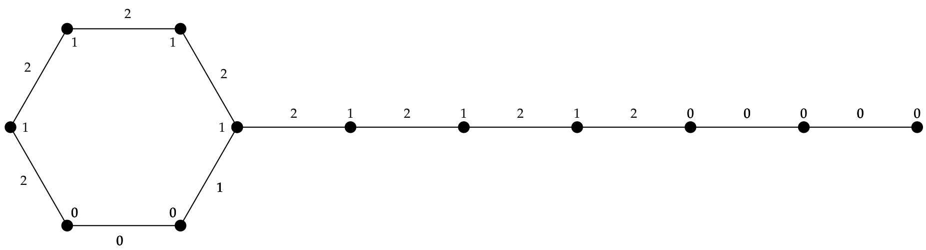

Figure 4. 3-TEPC labeling of $T_{6,6}$

Case 2 Let $p+q \equiv 1(\mod 3)$ which implies $p+q = 3t+1$, for $t \geq 3$. So for the given case, we need to discuss the following three subcases:

Case 2.1 If $p = 0(\mod 3)$ and $q \neq 0(\mod 3)$. We define $h : E_G \rightarrow \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p}{3}-1; \\ 2, & \text{if } \frac{p}{3} \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q-1}{3}; \\ 2, & \text{if } \frac{q-1}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

Case 2.2 If $p \neq 0(\mod 3)$ and $q = 0(\mod 3)$. We define $h : E_G \rightarrow \{0,1,2\}$ as:

$$g(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p-1}{3}-1; \\ 2, & \text{if } \frac{p-1}{3} \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q}{3}; \\ 2, & \text{if } \frac{q}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

Case 2.3 If $p \neq 0(\mod 3)$ and $q \neq 0(\mod 3)$. We define $h : E_G \rightarrow \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p-2}{3}; \\ 2, & \text{if } \frac{p-2}{3}+1 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q-2}{3}; \\ 2, & \text{if } \frac{q-2}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

So we obtain $sum(0) = 2t$, $sum(1) = sum(2) = 2t+1$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling as discussed in Figure 5.

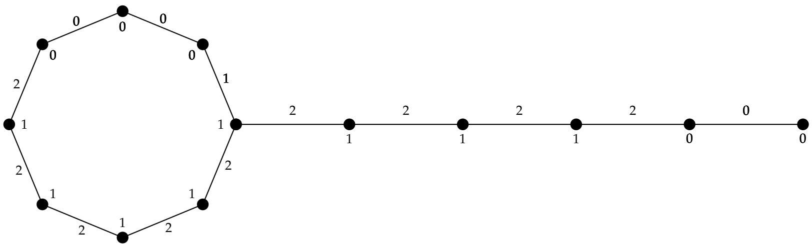

Figure 5. 3-TEPC labeling of $T_{8,5}$

Case 3 Let $p+q \equiv 2(\bmod 3)$ which implies $p+q=3t+2$, for $t \geq 3$. So for the given case, we need to discuss the following three subcases:

Case 3.2 If $p = 0(\bmod 3)$ and $q \neq 0(\bmod 3)$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p}{3}; \\ 2, & \text{if } \frac{p}{3}+1 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q-2}{3}; \\ 2, & \text{if } \frac{q-2}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

Case 3.2 If $p \neq 0(\bmod 3)$ and $q = 0(\bmod 3)$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p-2}{3}; \\ 2, & \text{if } \frac{p-2}{3}+1 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q}{3}; \\ 2, & \text{if } \frac{q}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

Case 3.3 If $p \neq 0(\bmod 3)$ and $q \neq 0(\bmod 3)$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{p-1}{3}; \\ 2, & \text{if } \frac{p-1}{3}+1 \leq j \leq p-1; \end{cases} \text{ and } h(u_pu_1) = 1.$$

$$h(v_jv_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq \frac{q-1}{3}; \\ 2, & \text{if } \frac{q-1}{3}+1 \leq j \leq q-1. \end{cases}, h(v_qu_p) = 2.$$

So we obtain $sum(0) = 2t+2$, $sum(1) = sum(2) = 2t+1$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling as discussed in Figure 6.

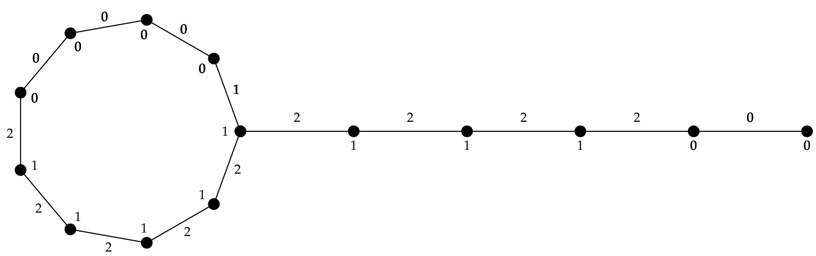

Figure 6. 3-TEPC labeling of $T_{9,5}$

□

Theorem 2. *The book graph B_q has 3-TEPC labeling.*

Proof. Let $V_G = \{u, v, u_j, v_j, 1 \leq j \leq q\}$ and $E_G = \{uu_j, 1 \leq j \leq q\} \cup \{vv_j, 1 \leq j \leq q\} \cup \{u_jv_j, 1 \leq j \leq q\} \cup \{uv\}$ as shown in Figure 2. Now we see the following three cases:

Case 1 Let $q \equiv 0 \pmod 3$ which implies $q = 3t$, for $t \geq 1$. We define $h : E_G \to \{0, 1, 2\}$ as:

Case 1.1 Now we define edge labeling if t is even:

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-1; \\ 1, & \text{if } t \leq j \leq 3t. \end{cases}$$

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq \frac{3t}{2}; \\ 1, & \text{if } \frac{3t}{2}+1 \leq j \leq 3t. \end{cases}, h(uv) = 2.$$

Case 1.2 Now we define edge labeling if t is odd:

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-1; \\ 1, & \text{if } t \leq j \leq 3t. \end{cases}$$

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq \frac{3t+1}{2}; \\ 1, & \text{if } \frac{3t+1}{2}+1 \leq j \leq 3t. \end{cases}, h(uv) = 1.$$

So we obtain $sum(0) = sum(1) = sum(2) = 5t + 1$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

Case 2 Let $q \equiv 1 \pmod 3$ which implies $q = 3t + 1$, for $t \geq 1$. We define $h : E_G \to \{0, 1, 2\}$ as:

Case 2.1 Now we define edge labeling if t is even:

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+1. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t+1. \end{cases}$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq \frac{3t}{2}; \\ 1, & \text{if } \frac{3t}{2}+1 \leq j \leq 3t+1. \end{cases}, h(uv) = 2.$$

Case 2.2 Now we define edge labeling if t is odd:

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+1. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t+1. \end{cases}$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq \frac{3t+1}{2}; \\ 1, & \text{if } \frac{3t+1}{2}+1 \leq j \leq 3t+1. \end{cases}, h(uv) = 1.$$

So we obtain $sum(0) = 5t + 2$, $sum(1) = sum(2) = 5t + 3$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

Case 3 Let $q \equiv 2 \pmod 3$ which implies $q = 3t + 2$, for $t \geq 1$. We define $h : E_G \to \{0, 1, 2\}$ as:

Case 3.1 Now we define edge labeling if t is even:

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+2. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t+2. \end{cases}$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 2, & \text{if } t+2 \leq j \leq \frac{3t}{2}+1; \\ 1, & \text{if } \frac{3t}{2}+2 \leq j \leq 3t+2. \end{cases}, h(uv) = 0.$$

Case 3.2 Now we define edge labeling if t is odd:

$$h(vv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+2. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 1, & \text{if } t+2 \leq j \leq 3t+2. \end{cases}$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq \frac{3t+1}{2}; \\ 1, & \text{if } \frac{3t+1}{2}+1 \leq j \leq 3t+2. \end{cases}, h(uv) = 2.$$

So we obtain $sum(0) = 5t+5$, $sum(1) = sum(2) = 5t+4$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

□

Example 1. The graph B_6 and its given 3-TEPC labeling as shown in Figure 7.

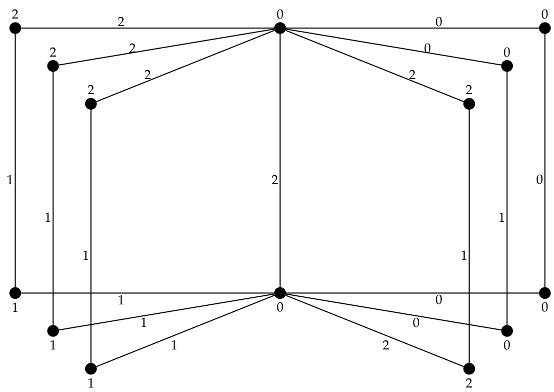

Figure 7. 3-TEPC labeling of B_6

Theorem 3. *The flower graph Fl_q has 3-TEPC labeling.*

Proof. Let $V_G = \{u, u_j, v_j, 1 \leq j \leq q\}$ and $E_G = \{uu_j, 1 \leq j \leq q\} \cup \{u_ju_{j+1}, 1 \leq j \leq q-1\} \cup \{u_jv_j, 1 \leq j \leq q\} \cup \{u_{j+1}v_j, 1 \leq j \leq q-1\} \cup \{u_qu_1\} \cup \{u_1v_q\}$ as shown in Figure 3. Now we see the following three cases:

Case 1 Let $q \equiv 0 \pmod 3$ which implies $q = 3t$, for $t \geq 1$. We define $h : E_G \to \{0, 1, 2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t-1; \end{cases} \text{ and } h(u_{3t}u_1) = 1.$$

$$h(u_{j+1}v_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-1; \\ 2, & \text{if } t \leq j \leq 3t-1; \end{cases} \text{ and } h(u_1v_{3t}) = 2.$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 2, & \text{if } t+2 \leq j \leq 2t; \\ 1, & \text{if } 2t+1 \leq j \leq 3t. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t-1; \\ 2, & \text{if } t \leq j \leq 3t. \end{cases}$$

So we obtain $sum(1) = 6t+1$, $sum(0) = sum(2) = 5t$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

Case 2 Let $q \equiv 1 \pmod 3$ which implies $q = 3t+1$, for $t \geq 1$. We define $h : E_G \to \{0, 1, 2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t; \end{cases} \text{ and } h(u_ju_{3t+1}) = 1.$$

$$h(u_{j+1}v_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t; \end{cases} \text{ and } h(u_1v_{3t+1}) = 2.$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 2, & \text{if } t+2 \leq j \leq 2t+1; \\ 1, & \text{if } 2t+2 \leq j \leq 3t+1. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+1. \end{cases}$$

So we obtain $sum(0) = 6t+3$, $sum(1) = sum(2) = 5t+2$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

Case 3 Let $q \equiv 2 \pmod 3$ which implies $q = 3t+2$, for $t \geq 1$. We define $h : E_G \to \{0,1,2\}$ as:

$$h(u_ju_{j+1}) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 2, & \text{if } t+1 \leq j \leq 3t+1; \end{cases} \text{ and } h(u_{3t+2}u_1) = 2.$$

$$h(u_{j+1}v_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t; \\ 1, & \text{if } t+1 \leq j \leq 3t+1; \end{cases} \text{ and } h(u_1v_{3t+2}) = 1.$$

$$h(uu_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 2, & \text{if } t+2 \leq j \leq 3t+2. \end{cases}, h(u_jv_j) = \begin{cases} 0, & \text{if } 1 \leq j \leq t+1; \\ 1, & \text{if } t+2 \leq j \leq 3t+2. \end{cases}$$

So we obtain $sum(0) = 6t+5$, $sum(1) = sum(2) = 5t+4$. Thus $|sum(x_1) - sum(x_2)| \leq 1$ for $0 \leq x_1 < x_2 \leq 2$. Hence h is 3-TEPC labeling.

□

Example 2. The graph Fl_6 and its given 3-TEPC labeling as shown in Figure 8.

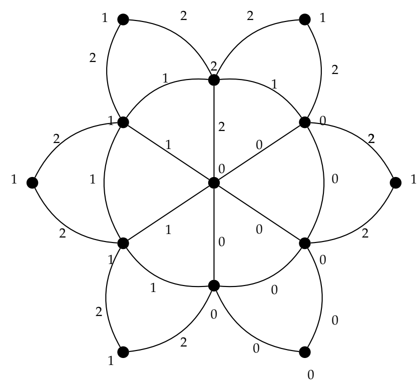

Figure 8. 3-TEPC labeling of Fl_6

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