Article

Development of a new numerical scheme for the solution of exponential growth and decay models

S. E. Fadugba

Department of Mathematics, Faculty of Science, Ekiti State University, Ado Ekiti, Nigeria.; sunday.fadugba@eksu.edu.ng

Abstract: This paper presents the development of a new numerical scheme for the solution of exponential growth and decay models emanated from biological sciences. The scheme has been derived via the combination of two interpolants namely, polynomial and exponential functions. The analysis of the local truncation error of the derived scheme is investigated by means of the Taylor's series expansion. In order to test the performance of the scheme in terms of accuracy in the context of the exact solution, four biological models were solved numerically. The absolute error has been computed successfully at each mesh point of the integration interval under consideration. The numerical results generated via the scheme agree with the exact solution and with the fifth order convergence based upon the analysis carried out. Hence, the scheme is found to be of order five, accurate and is a good approach to be included in the class of linear explicit numerical methods for the solution of initial value problems in ordinary differential equations.

Keywords: Accuracy, biological model, ordinary differential equation, initial value problem, local truncation error.

MSC: 34A12, 65L05, 65L70.

1. Introduction

Differential equations are useful to modern science and engineering. Differential equation models are used extensively in biology to study biochemical reactions, population dynamics, organism growth, and the spread of diseases. The most common use of differential equations in science is to model dynamical systems. Such physical models represent future estimation for any real world situation based on the data available in the past and present as detailed in [1–9].

However, these models are said to have no closed form solution in most of the real cases. In such situations, one has to compromise at numerical approximate solutions of the models achievable by various numerical techniques of different characteristics [10]. Development of new numerical integration methods with varying characteristics for the solution of initial value problems in ordinary differential equations has attracted the attention of many researchers in past and recent years as detailed in [11–17].

The main aim of the paper is to develop a new numerical method of order five via the combination of two interpolants for its possible acceptance within the class of linear explicit numerical techniques. Also the local truncation errors and order of accuracy of the scheme were thoroughly investigated. The rest of the paper is structured as follows; Section 2 presents the development of the scheme. In Section 3, the local truncation error of the scheme has been investigated. Also the order of accuracy of the scheme is obtained. Section 4 presents numerical experiments, discussion of results and concluding remarks.

2. Development of a new fifth order scheme

Consider an interpolating function of the form

$$F(x) = \sum_{j=0}^{5} \beta_j x^j + \beta_6 e^c, \tag{1}$$

Open J. Math. Sci. **2021**, 5, 18-26

19

where $\beta_0, \beta_2, \beta_3, ..., \beta_6$ are undetermined constants and c is a constant. The integration interval of $[a, b]$ is defined as

$$a = x_0 \le x \le x_n = b. \tag{2}$$

The step length is defined as

$$h = \frac{b - a}{N}. \tag{3}$$

The mesh point is defined as

$$x_n = x_0 + nh, n = 1, 2, .., N, \tag{4}$$

or

$$x_{n+1} = x_0 + (n + 1)h, n = 0, 1, 2, ..., N - 1. \tag{5}$$

Expanding (1) at the points x_n and x_{n+1} yields

$$F(x_n) = \sum_{j=1}^{5} \beta_j x_n^j + \beta_6 e^c, \tag{6}$$

and

$$F(x_{n+1}) = \sum_{j=1}^{5} \beta_j x_{n+1}^j + \beta_6 e^c \tag{7}$$

respectively. Differentiating (6) five times and using the fact that

$$F'(x_n) = f_n, F''(x_n) = f_n^{(1)}, F'''(x_n) = f_n^{(2)}, F^{(iv)}(x_n) = f_n^{(3)}, F^{(v)}(x_n) = f_n^{(4)}$$

yields

$$f_n = \sum_{j=1}^{5} j\beta_j x_n^{j-1}, \tag{8}$$

$$f_n^{(1)} = \sum_{j=2}^{5} j(j-1)\beta_j x_n^{j-2}, \tag{9}$$

$$f_n^{(2)} = \sum_{j=3}^{5} j(j-1)(j-2)\beta_j x_n^{j-3}, \tag{10}$$

$$f_n^{(3)} = \sum_{j=4}^{5} j(j-1)(j-2)(j-3)\beta_j x_n^{j-4}, \tag{11}$$

$$f_n^{(4)} = \sum_{j=5}^{5} j(j-1)(j-2)(j-3)(j-4)\beta_j x_n^{j-5}. \tag{12}$$

Equations (8) - (12) form a system of linear equations $AX = b$ which can be represented in a matrix form as follows;

$$\begin{bmatrix} 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 \\ 0 & 0 & 6 & 24x_n & 60x_n^2 \\ 0 & 0 & 0 & 24 & 120x_n \\ 0 & 0 & 0 & 0 & 120 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix} = \begin{bmatrix} f_n \\ f_n^{(1)} \\ f_n^{(2)} \\ f_n^{(3)} \\ f_n^{(4)} \end{bmatrix}, \tag{13}$$

where

$$A = \begin{bmatrix} 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 \\ 0 & 0 & 6 & 24x_n & 60x_n^2 \\ 0 & 0 & 0 & 24 & 120x_n \\ 0 & 0 & 0 & 0 & 120 \end{bmatrix}, X = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix}, b = \begin{bmatrix} f_n \\ f_n^{(1)} \\ f_n^{(2)} \\ f_n^{(3)} \\ f_n^{(4)} \end{bmatrix}.$$

Open J. Math. Sci. **2021**, 5, 18-26

20

Solving (13) by means of the Gauss Jordan method yields

$$\beta_1 = \frac{1}{24}\left(24f_n - 24x_n f_n^{(1)} + 12x_n^2 f_n^{(2)} - 4x_n^3 f_n^{(3)} + x_n^4 f_n^{(4)}\right), \tag{14}$$

$$\beta_2 = \frac{1}{12}\left(6f_n^{(1)} - 6x_n f_n^{(2)} + 3x_n^2 f_n^{(3)} - x_n^3 f_n^{(4)}\right), \tag{15}$$

$$\beta_3 = \frac{1}{12}\left(2f_n^{(2)} - 2x_n f_n^{(3)} + x_n^2 f_n^{(4)}\right), \tag{16}$$

$$\beta_4 = \frac{1}{24}\left(f_n^{(3)} - x_n f_n^{(4)}\right), \tag{17}$$

$$\beta_5 = \frac{1}{120}f_n^{(4)}. \tag{18}$$

Subtracting (6) from (7) yields

$$F(x_{n+1}) - F(x_n) = \sum_{j=1}^{5} \beta_j (x_{n+1}^j - x_n^j). \tag{19}$$

Therefore, (19) becomes

$$F(x_{n+1}) - F(x_n) = \beta_1(x_{n+1} - x_n) + \beta_2(x_{n+1}^2 - x_n^2) + \beta_3(x_{n+1}^3 - x_n^3) + \beta_4(x_{n+1}^4 - x_n^4) + \beta_5(x_{n+1}^5 - x_n^5). \tag{20}$$

Using (4) and (5) with $x_0 = 0$, one obtains the following

$$x_n = nh, \tag{21}$$
$$x_{n+1} = (n+1)h, \tag{22}$$
$$x_{n+1} - x_n = h, \tag{23}$$
$$x_{n+1}^2 - x_n^2 = (2n+1)h^2, \tag{24}$$
$$x_{n+1}^3 - x_n^3 = (3n^2 + 3n + 1)h^3, \tag{25}$$
$$x_{n+1}^4 - x_n^4 = (4n^3 + 6n^2 + 4n + 1)h^4, \tag{26}$$
$$x_{n+1}^5 - x_n^5 = (5n^4 + 10n^3 + 10n^2 + 5n + 1)h^5. \tag{27}$$

Using (21), then (14), (15), (16), (17) and (18) become

$$\beta_1 = \frac{1}{24}\left(24f_n - 24nh f_n^{(1)} + 12n^2h^2 f_n^{(2)} - 4n^3h^3 f_n^{(3)} + n^4h^4 f_n^{(4)}\right), \tag{28}$$

$$\beta_2 = \frac{1}{12}\left(6f_n^{(1)} - 6nh f_n^{(2)} + 3n^2h^2 f_n^{(3)} - n^3h^3 f_n^{(4)}\right), \tag{29}$$

$$\beta_3 = \frac{1}{12}\left(2f_n^{(2)} - 2nh f_n^{(3)} + n^2h^2 f_n^{(4)}\right), \tag{30}$$

$$\beta_4 = \frac{1}{24}\left(f_n^{(3)} - nh f_n^{(4)}\right), \tag{31}$$

$$\beta_5 = \frac{1}{120}f_n^{(4)}, \tag{32}$$

respectively. Since one-step numerical method shall be derived, let

$$y_{n+1} - y_n \equiv F(x_{n+1}) - F(x_n). \tag{33}$$

Substituting (20), (23)-(32) into (33), yields

$$y_{n+1} - y_n = \frac{h}{24}\left(24f_n - 24nh f_n^{(1)} + 12n^2h^2 f_n^{(2)} - 4n^3h^3 f_n^{(3)} + n^4h^4 f_n^{(4)}\right)$$
$$+ \frac{h^2}{12}\left(6f_n^{(1)} - 6nh f_n^{(2)} + 3n^2h^2 f_n^{(3)} - n^3h^3 f_n^{(4)}\right)(2n+1) + \frac{h^3}{12}\left(2f_n^{(2)} - 2nh f_n^{(3)} + n^2h^2 f_n^{(4)}\right)(3n^2 + 3n + 1)$$
$$+ \frac{h^4}{24}\left(f_n^{(3)} - nh f_n^{(4)}\right)(4n^3 + 6n^2 + 4n + 1) + \frac{h^5}{120}f_n^{(4)}(5n^4 + 10n^3 + 10n^2 + 5n + 1). \tag{34}$$

Setting

$$S_1 = \frac{1}{2}\left(24f_n - 24nhf_n^{(1)} + 12n^2h^2f_n^{(2)} - 4n^3h^3f_n^{(3)} + n^4h^4f_n^{(4)}\right), \tag{35}$$

$$S_2 = h\left(6f_n^{(1)} - 6nhf_n^{(2)} + 3n^2h^2f_n^{(3)} - n^3h^3f_n^{(4)}\right)(2n+1), \tag{36}$$

$$S_3 = h^2\left(2f_n^{(2)} - 2nhf_n^{(3)} + n^2h^2f_n^{(4)}\right)(3n^2 + 3n + 1), \tag{37}$$

$$S_4 = \frac{h^3}{2}\left(f_n^{(3)} - nhf_n^{(4)}\right)(4n^3 + 6n^2 + 4n + 1), \tag{38}$$

$$S_5 = \frac{h^4}{10}f_n^{(4)}(5n^4 + 10n^3 + 10n^2 + 5n + 1). \tag{39}$$

Therefore,

$$y_{n+1} = y_n + \frac{h}{12}(S_1 + S_2 + S_3 + S_4 + S_5). \tag{40}$$

Equation (40) is the newly developed one step scheme.

3. Order of accuracy of the new scheme

According to [18], local truncation error measures the order of accuracy of any numerical method. Consider the Taylor's series expansion of the form

$$\begin{aligned}y(x_n + h) =& y(x_n) + hf(x_n, y(x_n)) + \frac{h^2}{2}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) \\ &+ \frac{h^4}{4!}f^{(3)}(x_n, y(x_n)) + \frac{h^5}{5!}f^{(4)}(x_n, y(x_n)) + O(h^6).\end{aligned} \tag{41}$$

The local truncation error for the explicit one step method is given by

$$\tau_{n+1} = y(x_n + h) - y_{n+1}. \tag{42}$$

Substituting (40) and (41) into (42), yields

$$\begin{aligned}\tau_{n+1} =& y(x_n) + hf(x_n, y(x_n))\frac{h^2}{2}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) + \frac{h^4}{4!}f^{(3)}(x_n, y(x_n)) \\ &+ \frac{h^5}{5!}f^{(4)}(x_n, y(x_n)) + O(h^6) - y_n - \frac{h}{4!}(S_1 + S_2 + S_3 + S_4 + S_5).\end{aligned} \tag{43}$$

Substituting (35-39) into (43), one obtains

$$\begin{aligned}\tau_{n+1} =& y(x_n) + hf(x_n, y(x_n)) + \frac{h^2}{2}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) + \frac{h^4}{4!}f^{(3)}(x_n, y(x_n)) \\ &+ \frac{h^5}{5!}f^{(4)}(x_n, y(x_n)) + O(h^6) - y_n - \frac{h}{24}\left(24f_n - 24nhf_n^{(1)} + 12n^2h^2f_n^{(2)} - 4n^3h^3f_n^{(3)} + n^4h^4f_n^{(4)}\right) \\ &- \frac{h^2}{12}\left(6f_n^{(1)} - 6nhf_n^{(2)} + 3n^2h^2f_n^{(3)} - n^3h^3f_n^{(4)}\right)(2n+1) \\ &- \frac{h^3}{12}\left(2f_n^{(2)} - 2nhf_n^{(3)} + n^2h^2f_n^{(4)}\right)(3n^2 + 3n + 1) \\ &- \frac{h^4}{24}\left(f_n^{(3)} - nhf_n^{(4)}\right)(4n^3 + 6n^2 + 4n + 1) - \frac{h^5}{120}f_n^{(4)}(5n^4 + 10n^3 + 10n^2 + 5n + 1).\end{aligned} \tag{44}$$

Solving further, (44) becomes

$$\begin{aligned}\tau_{n+1} =& y(x_n) + hf(x_n, y(x_n)) + \frac{h^2}{2}f^{(1)}(x_n, y(x_n)) + \frac{h^3}{3!}f^{(2)}(x_n, y(x_n)) + \frac{h^4}{4!}f^{(3)}(x_n, y(x_n)) \\ &+ \frac{h^5}{5!}f^{(4)}(x_n, y(x_n)) + O(h^6) - \left[y_n + \frac{h}{5!}\left(120f_n + 60hf_n^{(1)} + 20h^2f_n^{(2)} + 5h^3f_n^{(3)} + h^4f_n^{(4)}\right)\right]. \tag{45}\end{aligned}$$

By means of localizing assumption, the terms up to h^5 have been canceled, thus the local truncation of the scheme is obtained as

$$\tau_{n+1} = O(h^6). \tag{46}$$

Equation (46) shows that the order of the scheme is five.

Remark 1. The local truncation error of the scheme is summarized in the following result.

Theorem 1. *By means of the Taylor's series expansion and the localizing assumption, the scheme given by (40) has fifth order accuracy.*

4. Numerical experiments, discussion of results and concluding remarks

The developed scheme was derived via the combination of two interpolants namely polynomial function and trigonometric function via MAPLE 18. The scheme (40) was implemented on biological models with the aid of MATLAB R2014a (8.3.0.532), 32-bit (Win 32) programming language.

4.1. Numerical experiments

Biological models that find applications in science in terms of modelling growth and decay shall be considered. The scheme (40) is implemented on these models and the results obtained were compared with the exact solutions.

Experiment 1. *Assume that a colony of 100 bacteria is multiplying at the rate of $k = 0.02$ per hour per individual. How many bacteria are there after 120 minutes? It is assumed that the colony grows continuously and without restriction.*

Here, it is evident that the rate of population growth is proportional to the size of population. It is possible to model this exponential growth with an initial value problem of first order ordinary differential equation of the form

$$\frac{dp}{dt} = kp, p(0) = 100, k = 0.02, 0 \le t \le 120, \tag{47}$$

where $p = p(t)$ is the population of the bacteria, $p(0) = p_0$ is the initial population of the bacteria, t is the time and k is the growth rate. The exact solution of (47) is obtained as

$$p(t) = 100 \exp(0.02t). \tag{48}$$

The comparative results analysis of the scheme (40) "p_n" and the exact solution "$p(t_n)$" with $h = 10$ are shown in Table 1.

Table 1. The results generated via the scheme (40) and the exact solution

| n	t_n	p_n	$p(t_n)$	$e_n = \lvert p(t_n) - p_n \rvert$
0.00	0.00	100.0000000000	100.0000000000	0.0000000000
1.00	10.00	122.1402666667	122.1402758160	0.0000091494
2.00	20.00	149.1824474140	149.1824697641	0.0000223501
3.00	30.00	182.2118390914	182.2118800391	0.0000409477
4.00	40.00	222.5540261644	222.5540928492	0.0000666848
5.00	50.00	271.8280810347	271.8281828459	0.0001018113
6.00	60.00	332.0115430506	332.0116922737	0.0001492230
7.00	70.00	405.5197840461	405.5199966845	0.0002126383
8.00	80.00	495.3029456200	495.3032424395	0.0002968195
9.00	90.00	604.9643385882	604.9647464413	0.0004078531
10.00	100.00	738.9050563898	738.9056098931	0.0005535033
11.00	110.00	902.5006062880	902.5013499434	0.0007436554
12.00	120.00	1102.3166471884	1102.3176380642	0.0009908757
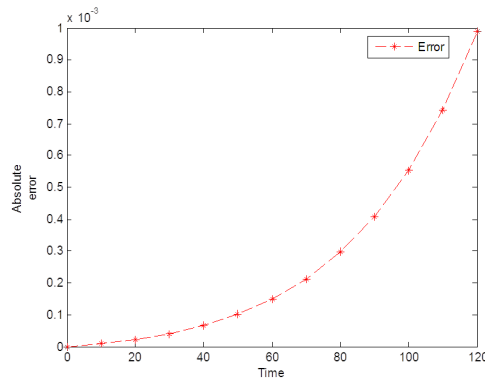
Figure 1. Errors generated via the scheme (40)

Experiment 2. *Consider the exponential decay model of the form*

$$\frac{dp}{dt} = -rp, p(0) = 100, r = 0.02, 0 \le t \le 120,$$ (49)

where $p = p(t)$ is the population of the bacteria, $p(0) = p_0$ is the initial population of the bacteria, t is the time and r is the decay rate. The exact solution of (49) is obtained as

$$p(t) = 100 \exp(-0.02t).$$ (50)

　　The comparative results analysis of the scheme (40) "p_n" and the exact solution "$p(t_n)$" with $h = 10$ are shown in Table 2.

Table 2. The results generated via the scheme (40) and the exact solution

| n	t_n	p_n	$p(t_n)$	$e_n = |p(t_n) - p_n|$
0.00	0.00	100.0000000000	200.0000000000	0.0000000000
1.00	10.00	81.8730666667	81.8730753078	0.0000086411
2.00	20.00	67.0319904540	67.0320046036	0.0000141495
3.00	30.00	54.8811462324	54.8811636094	0.0000173770
4.00	40.00	44.9328774423	44.9328964117	0.0000189694
5.00	50.00	36.7879247036	36.7879441171	0.0000194135
6.00	60.00	30.1194021179	30.1194211912	0.0000190734
7.00	70.00	24.6596781756	24.6596963942	0.0000182186
8.00	80.00	20.1896347525	20.1896517995	0.0000170470
9.00	90.00	16.5298731206	16.5298888222	0.0000157015
10.00	100.00	13.5335140400	13.5335283237	0.0000142837
11.00	110.00	11.0803029723	11.0803158362	0.0000128639
12.00	120.00	9.0717838394	9.0717953289	0.0000114896
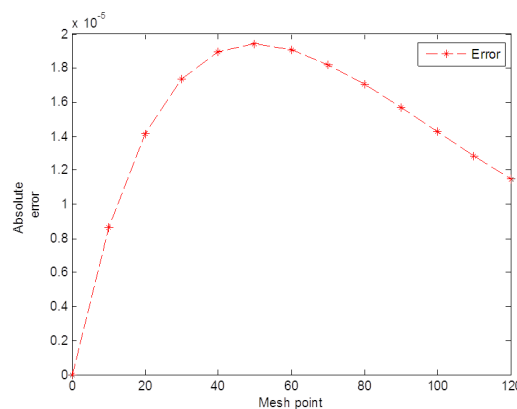
Figure 2. Errors generated via the scheme (40)

Experiment 3. [19] *Suppose there are 1000 birds on an Island, breeding with a constant continuous growth rate of 10% per year. But now birds migrate to the Island at a constant rate of 100 new arrivals per year. How many birds are on the Island after seven years?*

Let $p = p(t)$ be the number of birds on Island, t is the time, k is a constant continuous growth rate, m be the rate of migration of the population. The model equation for population growth with migration is given by

$$\frac{dp}{dt} = kp + m, p(0) = 1000, k = 0.1, m = 100, 0 \leq t \leq 7. \tag{51}$$

The exact solution of (51) is obtained as

$$p(t) = 2000 \exp(0.1t) - 1000. \tag{52}$$

The comparative results analysis of the scheme (40) "p_n" and the exact solution "$p(t_n)$" with $h = 0.5$ are shown in Table 3.

Table 3. The results generated via the scheme (40) and the exact solution

| n	t_n	p_n	$p(t_n)$	$e_n = \|p(t_n) - p_n\|$
0.00	0.00	1000.0000000000	1000.0000000000	0.0000000000
1.00	0.50	1102.5421927083	1102.5421927520	0.0000000437
2.00	1.00	1210.3418360594	1210.3418361513	0.0000000919
3.00	1.50	1323.6684853116	1323.6684854566	0.0000001449
4.00	2.00	1442.8055161172	1442.8055163203	0.0000002032
5.00	2.50	1568.0508331085	1568.0508333755	0.0000002670
6.00	3.00	1699.7176148152	1699.7176151520	0.0000003368
7.00	3.50	1838.1350967735	1838.1350971865	0.0000004131
8.00	4.00	1983.6493947863	1983.6493952825	0.0000004963
9.00	4.50	2136.6243703934	2136.6243709803	0.0000005869
10.00	5.00	2297.4425407147	2297.4425414003	0.0000006856
11.00	5.50	2466.5060349420	2466.5060357348	0.0000007928
12.00	6.00	2644.2375998718	2644.2376007810	0.0000009092
13.00	6.50	2831.0816569923	2831.0816580278	0.0000010355
14.00	7.00	3027.5054137686	3027.5054149410	0.0000011723
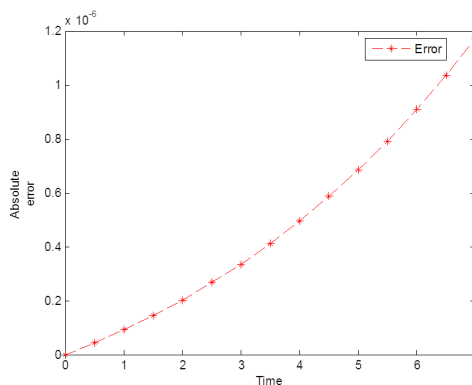
Figure 3. Errors generated via the scheme (40)

Experiment 4. [19] *A cell culture in a biology laboratory currently holds 1000000 cells. The cells have a constant continuous birth rate of 1.5% and death rate of 0.5% per hour. Cells are extracted from the culture for an experiment at the rate of 5000 per hour. How many cells will be in the culture 1 hour from now?*

Let $p = p(t)$ be the population, t be the number of hours from now, k be the difference between the birth rate and death rate, b be the birth rate, d be the death rate and n be the extracted cells. The model equation for the growth and decay with input and output is given by

$$\frac{dp}{dt} = (b - d)p + n, p(0) = 1000000, 0 \leq t \leq 1, \tag{53}$$

where $b = 0.015, d = -0.005, k = 0.01, n = -5000$. *The exact solution of* (53) *is obtained as*

$$p(t) = 500000 \exp(0.01t) + 500000. \tag{54}$$

The comparative results analysis of the scheme (40) *"p_n" and the exact solution "$p(t_n)$" with $h = 0.1$ are shown in Table* 4.

Table 4. The results generated via the scheme (40) and the exact solution

| n	t_n	p_n	$p(t_n)$	$e_n = |p(t_n) - p_n|$
0.00	0.00	1000000.0000000000	1000000.0000000000	0.0000000000
1.00	0.10	1000500.2550850213	1000500.2500833542	0.0050016671
2.00	0.20	1001001.0106753386	1001001.0006670002	0.0100083384
3.00	0.30	1001502.2672717074	1001502.2522516886	0.0150200188
4.00	0.40	1002004.0253753845	1002004.0053386709	0.0200367136
5.00	0.50	1002506.2854881278	1002506.2604297005	0.0250584274
6.00	0.60	1003009.0481121978	1003009.0180270325	0.0300851653
7.00	0.70	1003512.3137503569	1003512.2786334243	0.0351169326
8.00	0.80	1004016.0829058710	1004016.0427521367	0.0401537343
9.00	0.90	1004520.3560825090	1004520.3108869339	0.0451955751
10.00	1.00	1005025.1337845444	1005025.0835420840	0.0502424604
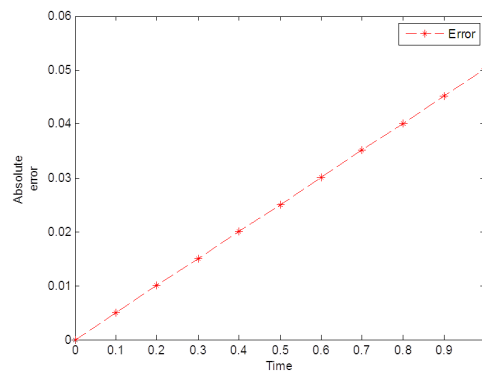
Figure 4. Errors generated via the scheme (40)

4.2. Summary of the results

The summary of the results generated via the scheme and the exact solution at the final mesh point time in the nearest whole number is presented as follows (Table 5):

Table 5. The summary of the accuracy of results generated via the scheme (40) against exact solution

Experiments	t_{nfl}	p_{nfl}	$p(t_{nfl})$	$e_{nfl} = |p(t_n fl) - p_{nfl}|$
Experiment 1	120 mins.	1102	1102	0.000
Experiment 2	120 mins.	9	9	0.0000
Experiment 3	7 hours	3027	3027	0.0000
Experiment 4	1 hour	1005025	1005025	0.0000

4.3. Discussion of results and concluding remarks

In this paper, a new fifth order scheme for the solution of initial value problems in ordinary differential equations emanated from biological sciences is developed via the combination of two interpolants. Four numerical experiments have been performed to test the performance of the scheme in terms of the accuracy in the context of the exact solution as shown in Tables 1-4 and also absolute errors computed at each mesh point of the integration interval under consideration as demonstrated in Figures 1-4. The numerical results in Tables 1-4 show that the fifth order scheme is accurate and converges faster to the exact solution. The effect of the constant k determines the solution of the models under consideration. It is observed from Tables 1, 3 and 4

that the results of the scheme and the exact solution increase exponentially over time. It is also observed from Table 2 that the results of both the scheme and the exact solution decrease over time. When compared with the exact solutions, the fifth order scheme yielded smaller amount of errors as seen from the above Figures 1-4. The summary of the results generated via the fifth order scheme in the context of the exact solution is presented in Table 5. Hence, the fifth order scheme is a good approach to be included in the class of linear explicit numerical methods as its analysis carried out agrees with the exact solution of exponential growth and decay models emanated biological sciences. Finally, all the calculations were carried out via MATLAB R2014a, Version: 8.3.0.552, 32 bit (Win 32) in double precision.

Conflicts of Interest: "The author declares no conflict of interest."

References

[1] Ansari, M. Y., Shaikh, A. A., & Qureshi, S. (2018). Error bounds for a numerical scheme with reduced slope evaluations. *Journal of Applied Environmental and Biological Sciences, 8*(7), 67-76.

[2] Bird, J. (2017). *Higher Engineering Mathematics.* Taylor and Francis Group, London.

[3] Butcher, J. C. (2016). *Numerical Methods for Ordinary Differential Equations.* John Wiley and Sons, Ltd., United Kingdom.

[4] Jain, M. K. (2003). *Numerical Methods for Scientific and Engineering Computation.* New Age International (P) Ltd., New Delhi, India.

[5] Lambert, J. D. (1991). *Numerical Methods for Ordinary Differential Systems: the Initial Value Problem.* John Wiley and Sons, Inc., New York.

[6] Lambert, J. D. (2011). *Computational Methods in Ordinary Differential Equations.* John Wiley and Sons Inc.

[7] Rabiei, F., & Ismail, F. (2011). Third-order Improved Runge-Kutta method for solving ordinary differential equation. *International Journal of Applied Physics and Mathematics, 1*(3), 191-194.

[8] *Applications of Differential Equations.* http://faculty.bard.edu/ belk/math213s14, accessed on April 27, 2020.

[9] Zill, D. G. (2012). *A First Course in Differential Equations with Modeling Applications.* Cengage Learning.

[10] Qureshi, S., & Emmanuel, F. S. (2018). Convergence of a numerical technique via interpolating function to approximate physical dynamical systems. *Journal of Advanced Physics, 7*(3), 446-450.

[11] Fadugba, S. E., & Idowu, J. O. (2019). Analysis of the properties of a third order convergence numerical method derived via transcendental function of exponential form. *International Journal of Applied Mathematics and Theoretical Physics, 5*(4), 97–103.

[12] Ogunrinde, R. B., & Fadugba, S. E. (2012). Development of a new scheme for the solution of initial value problems in ordinary differential equations. *International Organization of Scientific Research Journal of Mathematics (IOSRJM), 2*, 24–29.

[13] Fadugba, S. E., & Okunlola, J. T. (2017). Performance measure of a new one-step numerical technique via interpolating function for the solution of initial value problem of first order differential equation. *World Scientific News, 90*, 77–87.

[14] Fadugba, S. E., & Olaosebikan, T. E. (2018). Comparative study of a class of one-step methods for the numerical solution of some initial value problems in ordinary differential equations. *Research Journal of Mathematics and Computer Science, 2*(9), 1–11.

[15] Qureshi, S., Shaikh, A. A., & Chandio, M. S. (2019). A new iterative integrator for Cauchy problems. *Sindh University Research Journal-SURJ (Science Series), 45*(3).

[16] Rabiei, F., & Ismail, F. (2012). Fifth–order Improved Runge–Kutta method for solving ordinary differential equation. *Australian Journal of Basic and Applied Sciences, 6*(3), 97–105.

[17] Rabiei, F., Ismail, F., & Suleiman, M. (2013). Improved Runge–Kutta methods for solving ordinary differential equations. *Sains Malaysiana, 42*(11), 1679–1687.

[18] Fadugba, S. E. & Qureshi, S. (2019). Convergent numerical method using transcendental function of exponential type to solve continuous dynamical systems. *Punjab University Journal of Mathematics, 51*(10), 45–56.

[19] Brown, P., Evans, M., & Hunt, D. (2013). *Growth and decay–A guide for teachers* (Years 11-12). Educational Services, Australia.