Article

Super cyclic antimagic covering for some families of graphs

Muhammad Numan[1], Saad Ihsan Butt[2,*] and Amir Taimur[3]

[1] Department of Mathematics, COMSATS University Islamabad, Attock, Pakistan.
[2] Department of Mathematics, COMSATS University Islamabad, Lahore, Pakistan.
[3] Department of Mathematics, Islamia College Peshawar, KPK, Pakistan.
* Correspondence: saadihsanbutt@cuilahore.edu.pk

Abstract: Graph labeling plays an important role in different branches of sciences. It gives useable information in the study of radar, missile and rocket theory. In scheme theory, coding theory and computer networking graph labeling is widely employed. In the present paper, we find necessary conditions for the octagonal planner map and multiple wheel graph to be super cyclic antimagic cover and then discuss their super cyclic antimagic covering.

1. Introduction and definitions

In the present paper all graphs are finite simple and we follow the notion and terminology from the book [1]. For the cardinality of vertex set and edge set, we use n and m respectively. Graph labeling is a mapping which assigns labels to vertices or edges or both from the set of positive integers. Usually if we assign label to vertices then corresponding labeling called vertex labeling, similarly if we assign label to edges then that is called edge labeling and to assign label to both vertices and edges then such labeling called total labeling. The weight of a vertex under a labeling is the sum of labels assign to that vertex(if present) and label of edges(if present) incident to that vertex. The weight of an edge is the sum of labels assign to that edge(if present) and labels of end vertices(if present) of that edge. Throughout the paper we consider total labeling a bijective function from the set $V(G) \cup E(G)$ into the set $\{1, 2, 3, ..., n + m\}$. We give the following definitions of our interest:

Definition 1. [2] A total labeling f is called (a, d)-edge-antimagic total labeling if the set of all edge weights is $\{a, a + d, a + 2d, a + 3d, ..., a + (m-2)d, a + (m-1)d\}$, where $a > 0$ and $d \geq 0$ are two integers.

The (a, d)-edge-antimagic total labeling was first defined by Simanjuntak *et al.,* in [2]. If in such type of labeling we assign smallest possible labels to vertices then it is called super (a, d)-edge-antimagic total labeling. For the case of $d = 0$ the corresponding $(a, 0)$-edge-antimagic total labeling is edge-magic total labeling. In above definition the idea of edges weight is used, in particular we covered any graph by edges. Gutierrez and Llado in [3,4] used idea of covering and extended the theory of edge-magic total labeling into a magic total covering.

Definition 2. [3,4] For a graph G a family of subgraphs K_i, $1 \leq i \leq s$ is called an edge covering of G if every edge of G belong to some K_i. More precisely we say that G admit $(K_1, K_2, K_3, ..., K_s)$-covering. In the case when all K_i isomorphic to some given graph K then we called that G has K-covering.

Definition 3. [3,4] If we have a graph G which admits K-covering and f is a total mapping on graph G then weight of the subgraph K is denoted by $W(K)$ and is:

$$W(K) = \sum_{x \in V(K)} f(v) + \sum_{y \in E(K)} f(y).$$

If the weight set of all subgraphs K_i, $1 \leq i \leq s$ isomorphic to K is a given constant then we say that G admits K-magic total covering. When the label of vertices taken from the $\{1, 2, 3, ..., n\}$ then K-magic total covering called K-supermagic total covering.

Some results on K-supermagic total covering can be seen in [3–9]. Inayah *et al.*, [10] extend the theory of K-supermagic total covering to super (a, d)-K-antimagic total covering. Ali *et al.*, in [11] proposed Super Edge-Magicness of Stars Like Graphs and H-groupmagic total labelings of families of fangraphs in [12].

Definition 4. [10] For a given graph G the total mapping f is called (a, d)-K-antimagic total covering, if the weights of all subgraphs K_i isomorphic to K form and arithmetic sequence starts from a positive non-zero integer a and vary with difference $d \geq 0$ where d is also an integer. If vertices label from the set $\{1, 2, 3, ..., n\}$ then such labeling called super (a, d)-K-antimagic total covering.

In [10] the authors showed that shackle of p copies of any graph K can be covered by p subgraphs isomorphic to K is K-super antimagic. Recently, Fenovcikova *et al.*, [13] proved that wheels are cycle antimagic. Some more result on super (a, d)-K-antimagic labeling can be found in [14] but still this topic remains widely open. In the present paper, we find necessary conditions for the octagonal planner map and multiple wheel graph to be super cyclic antimagic cover and then discuss their super cyclic antimagic covering.

2. Super cyclic (a, d)-antimagic covering of octagonal planner map

The octagonal planner map is a plane graph which is constructed from an octagon. If we have octagons in the plane then we join these octagons in such a way that they cover the plane and called that octagonal planner map. In octagonal planner map the plane is covered by octagons and squares. Let l be the number of columns and k be the number of rows in octagonal planner map then we denote the octagonal planner map by O_l^k shown in Figure 1. The vertex set $V(O_l^k)$ and edge set $E(O_l^k)$ of octagonal planner maps are:

$V(O_k^l) = \{e_i^j, f_i^j, 1 \leq i \leq l, \ 1 \leq j \leq k+1\} \cup \{g_i^j, h_i^j, \ 1 \leq i \leq l+1, \ 1 \leq j \leq k\}$,

$E(O_k^l) = \{e_i^j f_i^j, \ 1 \leq i \leq l, \ 1 \leq j \leq k+1\} \cup \{g_i^j h_i^j, 1 \leq i \leq l+1, \ 1 \leq j \leq k\} \cup \{f_i^j g_{i+1}^j, e_i^j g_i^j, h_i^j e_i^{j+1}, f_i^{j+1} g_{i+1}^j, 1 \leq i \leq l, \ 1 \leq j \leq k\}$.

However the cardinality of $|V(O_k^l)| = 4kl + 2l + 2k$ and $|E(O_k^l)| = 6kl + l + k$. We can cover the octagonal
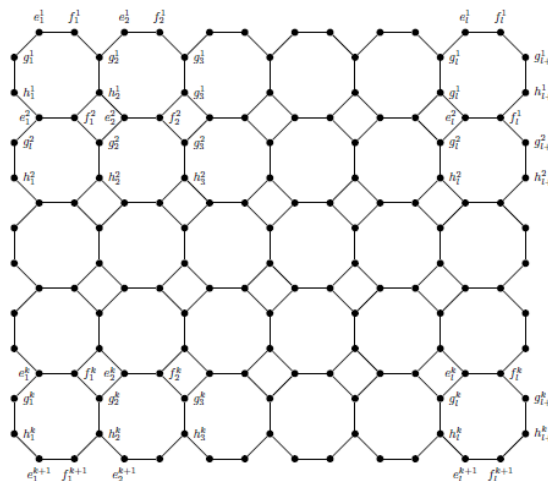

Figure 1. Octagonal planar map

planner map O_l^k by the 8-sided cycles $C_{8,i}^j$, for $1 \leq j \leq k, 1 \leq i \leq l$, having following vertex set and edge set:

$$V(C_{8,i}^j) = \{e_i^j, f_i^j, g_{i+1}^j, h_{i+1}^j, f_i^{j+1}, e_i^{j+1}, h_i^j, g_i^j; 1 \leq i \leq l, 1 \leq j \leq k\}$$

and

$$E(C_{8,i}^j) = \{e_i^j f_i^j, f_i^j g_{i+1}^j, g_{i+1}^j h_{i+1}^j, f_i^{j+1} h_{i+1}^j, e_i^{j+1} f_i^{j+1}, e_i^{j+1} h_i^j, g_i^j h_i^j, e_i^j g_i^j; 1 \leq i \leq l, 1 \leq j \leq k\}.$$

Open J. Math. Sci. **2021**, 5, 27-33

29

The upper bound for the parameter d under super (a, d)-C_8-antimagic covering of O_l^k is computed as following:

Theorem 1. *The parameter d satisfy following relation under a super (a, d)-C_8-antimagic covering of O_l^k*

$$d \leq 80 + \left\lceil \frac{24l + 24k - 48}{kl - 1} \right\rceil.$$

Proof. Let ϕ be a super (a, d)-C_8-antimagic covering of the graph O_l^k then minimum possible weight of C_8 is atleast

$$\sum_{i=1}^{8} i + \sum_{i=1}^{8} (4kl + 2l + 2k + i) = 32kl + 16l + 16k + 72.$$

From the above inequality we have,

$$a \geq 32kl + 16l + 16k + 72$$

and the largest weight of C_8 is no more than

$$\sum_{i=1}^{8} (4kl + 2l + 2k + 1 - i) + \sum_{i=1}^{8} (10kl + 3l + 3k + 1 - i) = 112kl + 40l + 40k - 56.$$

Thus

$$a + (kl - 1)d \leq 112kl + 40l + 40k - 56.$$

By using above two inequalities we get,

$$d \leq 80 + \left\lceil \frac{24l + 24k - 48}{kl - 1} \right\rceil.$$

For every $k \geq 25$ and $l \geq 25$, we get $d \leq 81$. $\quad\square$

Theorem 2. *The octagonal planner map O_l^k admits super (a, d)-C_8-antimagic covering for $d \in \{2, 4, 6, 8, 10, 12, 14, 16\}$, and $k, l \geq 2$.*

Proof. For every $d \in \{2, 4, 6, 8, 10, 12, 14, 16\}$ and for every $1 \leq i \leq l$, $1 \leq j \leq k$ define a total labeling ϕ_d of O_l^k as follow:

- $\phi_2\left(e_i^j g_i^j\right) = \phi_6\left(e_i^j g_i^j\right) = \phi_{10}\left(e_i^j g_i^j\right) = \phi_{14}\left(e_i^j g_i^j\right) = 3(k+1)l + 2k(l+1) + (j-1)l + i$,

- $\phi_4\left(e_i^j g_i^j\right) = \phi_8\left(e_i^j g_i^j\right) = \phi_{12}\left(e_i^j g_i^j\right) = \phi_{16}\left(e_i^j g_i^j\right) = 3(k+1)l + 2k(l+1) + (k-j)l + i$,

- $\phi_2\left(f_i^j g_{i+1}^j\right) = \phi_6\left(f_i^j g_{i+1}^j\right) = \phi_{10}\left(f_i^j g_{i+1}^j\right) = \phi_{14}\left(f_i^j g_{i+1}^j\right) = 3(k+1)l + 2k(l+1) + kl + (k+1-j)l + 1 - i$,

- $\phi_4\left(f_i^j g_{i+1}^j\right) = \phi_8\left(f_i^j g_{i+1}^j\right) = \phi_{12}\left(f_i^j g_{i+1}^j\right) = \phi_{16}\left(f_i^j g_{i+1}^j\right) = 3(k+1)l + 2k(l+1) + kl + (k-j)l + i$,

- $\phi_2\left(h_i^j e_i^{j+1}\right) = \phi_4\left(h_i^j e_i^{j+1}\right) = \phi_6\left(h_i^j e_i^{j+1}\right) = \phi_8\left(h_i^j e_i^{j+1}\right) = \phi_{10}\left(h_i^j e_i^{j+1}\right) = \phi_{12}\left(h_i^j e_i^{j+1}\right) = 3(k+1)l + 2k(l+1) + 2kl + (k+1-j)l + 1 - i$,

- $\phi_{14}\left(h_i^j e_i^{j+1}\right) = \phi_{16}\left(h_i^j e_i^{j+1}\right) = 3(k+1)l + 2k(l+1) + kl + (k-j)l + i$,

- $\phi_2\left(h_{i+1}^j f_i^{j+1}\right) = \phi_4\left(h_{i+1}^j f_i^{j+1}\right) = \phi_6\left(h_{i+1}^j f_i^{j+1}\right) = \phi_8\left(h_{i+1}^j f_i^{j+1}\right) = \phi_{10}\left(h_{i+1}^j f_i^{j+1}\right) = \phi_{12}\left(h_{i+1}^j f_i^{j+1}\right) = 3(k+1)l + 2k(l+1) + 3kl + (k+1-j)l + 1 - i$,

- $\phi_{14}\left(h_{i+1}^j f_i^{j+1}\right) = \phi_{16}\left(h_{i+1}^j f_i^{j+1}\right) = 3(k+1)l + 2k(l+1) + 3kl + (k-j)l + i$.

For every $1 \leq i \leq l$, $1 \leq j \leq k+1$

- $\phi_2\left(e_i^j\right) = \phi_4\left(e_i^j\right) = \phi_6\left(e_i^j\right) = \phi_8\left(e_i^j\right) = (j-1)l + i$,

Open J. Math. Sci. **2021**, 5, 27-33

30

- $\phi_{10}\left(e_i^j\right) = \phi_{12}\left(e_i^j\right) = \phi_{14}\left(e_i^j\right) = \phi_{16}\left(e_i^j\right) = (k-j)l+i,$

- $\phi_2\left(f_i^j\right) = \phi_4\left(f_i^j\right) = (k+1)l + (k+1-j)l+1-i,$

- $\phi_6\left(f_i^j\right) = \phi_8\left(f_i^j\right) = \phi_{10}\left(f_i^j\right) = \phi_{12}\left(f_i^j\right) = \phi_{14}\left(f_i^j\right) = \phi_{16}\left(f_i^j\right) = (k+1)l + (k-j)l+i,$

- $\phi_2(e_i^j f_i^j) = \phi_4(e_i^j f_i^j) = \phi_6(e_i^j f_i^j) = \phi_8(e_i^j f_i^j) = \phi_{10}(e_i^j f_i^j) = \phi_{12}(e_i^j f_i^j) = 2(k+1)l + 2k(l+1) + (j-1)l+i,$

- $\phi_{14}(e_i^j f_i^j) = \phi_{16}(e_i^j f_i^j) = 2(k+1)l + 2k(l+1) + (k+1-j)l+i.$

For every $1 \le i \le l+1,\ 1 \le j \le k$

- $\phi_2\left(g_i^j\right) = \phi_4\left(g_i^j\right) = 2(k+1)l + (j-1)(l+1)+i,$

- $\phi_6\left(g_i^j\right) = \phi_8\left(g_i^j\right) = \phi_{10}\left(g_i^j\right) = \phi_{12}\left(g_i^j\right) = \phi_{14}\left(g_i^j\right) = \phi_{16}\left(g_i^j\right) = 2(k+1)l + (k-j)(l+1)+i,$

- $\phi_2\left(h_i^j\right) = \phi_4\left(h_i^j\right) = \phi_6\left(h_i^j\right) = \phi_8\left(g_i^j\right) = 2(k+1)l + k(l+1) + (k+1-j)(l+1)+1-i,$

- $\phi_{10}\left(h_i^j\right) = \phi_{12}\left(h_i^j\right) = \phi_{14}\left(h_i^j\right) = \phi_{16}\left(h_i^j\right) = 2(k+1)l + k(l+1) + (k-j)(l+1)+i,$

- $\phi_d\left(g_i^j h_i^j\right) = 3(k+1)l + 2k(l+1) + 4kl + (j-1)(l+1)+i.$

The total labeling ϕ_d label the vertices e_i^j, f_i^j, g_i^j and h_i^j from the set $\{1,2,....,4kl+2l+2k\}$, where each label is used once and covers all the labels used. All the edges of the graph O_l^k labeled from the set $\{4kl+2l+2k+1, 4kl+2l+2k+2,....,10kl+3l+3k\}$, where each label used once and all the labels used from the mentioned set. This shows that the total labeling ϕ_d is a bijective function from the set $V(O_l^k) \bigcup E(O_l^k)$ into the set $\{1,2,....,10kl+3l+3k\}$.

Under the labeling ϕ_d weight of the cycles $C_{8,i}^j$, where $1 \le i \le l$ and $1 \le j \le k$ are:

$$wt_d(C_{8,i}^j) = \phi_d\left(e_i^j\right) + \phi_d\left(f_i^j\right) + \phi_d\left(g_{i+1}^j\right) + \phi_d\left(h_{i+1}^j\right) + \phi_d\left(f_i^{j+1}\right) + \phi_d\left(e_i^{j+1}\right) + \phi_d\left(h_i^j\right) + \phi_d\left(g_i^j\right) + \phi_d\left(e_i^j f_i^{j-1}\right)$$
$$+ \phi_d\left(f_i^j g_{i+1}^j\right) + \phi_d\left(g_{i+1}^j h_{i+1}^j\right) + \phi_d\left(h_{i+1}^j f_i^{j+1}\right) + \phi_d\left(e_i^{j+1} f_i^{j+1}\right) + \phi_d\left(h_i^j e_i^{j+1}\right) + \phi_d\left(g_i^j h_i^j\right) + \phi_d\left(g_i^j e_i^j\right).$$

$$wt_d(C_{8,i}^j) = \begin{cases} 71kl + 31l + 20k + 7 - 2lj + 2i, & for\ d=2, \\ 72kl + 31l + 20k + 6 - 4lj + 4i, & for\ d=4, \\ 73kl + 31l + 22k + 7 - 6lj + 6i, & for\ d=6, \\ 74kl + 31l + 22k + 6 - 8lj + 8i, & for\ d=8, \\ 75kl + 29l + 22k + 5 - 10lj + 10i, & for\ d=10, \\ 76kl + 29l + 22k + 4 - 12lj + 12i, & for\ d=12, \\ 74kl + 29l + 22k + 3 - 14lj + 14i, & for\ d=14, \\ 75kl + 29l + 22k + 2 - 16lj + 16i, & for\ d=16. \end{cases}$$

For the cases $d = \{2,4,6,8,10,12,14,16\}$, when we put values of i and j from $1 \le i \le l$ and $1 \le j \le k$ in $wt_d(C_{8,i}^j)$, we get the sets $\{69kl + 31l + 20k + 9, 69kl + 31l + 20k + 11, 69kl + 31l + 20k + 13, ..., 71kl + 31l + 20k + 7\}$, $\{68kl + 31l + 20k + 10, 68kl + 31l + 20k + 14, 68kl + 31l + 20k + 18, ..., 72kl + 31l + 20k + 6\}$, $\{67kl + 31l + 22k + 13, 67kl + 31l + 22k + 19, 67kl + 31l + 22k + 25, ..., 73kl + 31l + 22k + 7\}$, $\{66kl + 31l + 22k + 14, 66kl + 31l + 22k + 22, 66kl + 31l + 22k + 30, ..., 74kl + 31l + 22k + 6\}$, $\{65kl + 29l + 22k + 15, 65kl + 29l + 22k + 25, 65kl + 29l + 22k + 35, ..., 75kl + 29l + 22k + 5\}$, $\{64kl + 29l + 22k + 16, 66kl + 29l + 22k + 28, 64kl + 29l + 22k + 40, ..., 76kl + 29l + 22k + 4\}$, $\{60kl + 29l + 22k + 17, 60kl + 29l + 22k + 31, 60kl + 29l + 22k + 45, ..., 74kl + 29l + 22k + 3\}$ and $\{59kl + 29l + 22k + 18, 59kl + 29l + 22k + 34, 59kl + 29l + 22k + 50, ..., 75kl + 29l + 22k + 2\}$ which showed that weights of the cycles $C_{8,i}^j$ form an arithmetic sequence with differences $2,4,6,8,10,12,14$ and 16.

$\square$

3. Super (a, d)-C_3-antimagic covering of multiple-wheel graph mW_n

A multiple-wheel graph mW_n of size n can be composed of $mC_n + K_1$, that is it consists of m cycles of size n, where the vertices of the m cycles are all connected to a common hub. For $m = 2$ the multiple-wheel graph is called double-wheel graph, the graceful labeling of double-wheel graph were discussed in [15]. Let the vertex set and edge set of mW_n be define as follows:

$$V(mW_n) = \{u_i^j;\ 0 \leq i \leq n-1,\ 0 \leq j \leq m-1\} \cup \{c\},$$
$$E(mW_n) = \{cu_i^j; 0 \leq i \leq n-1,\ 0 \leq j \leq m-1\} \cup \{u_i^j u_{i+1}^j;\ 0 \leq i \leq n-1, 0 \leq j \leq m-1\},$$

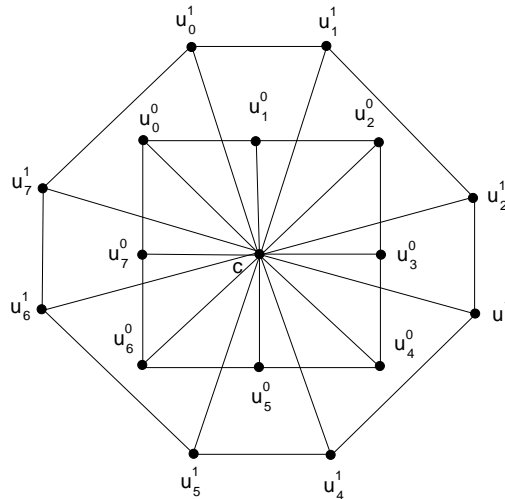where $|V(mW_n)| = 1 + mn$ and $|E(mW_n)| = 2mn$ be the cardinality of vertex set and edge set respectively.

Figure 2. Multiple wheel graph $2W_8$

The multiple-wheel graph mW_n shown in Figure 2 can be cover with cycle C_3. We denote the cycles C_3 by $C_{3,i}^j$ having vertex set and edge set as follows:

$$V(C_{3,i}^j) = \{c, u_i^j, u_{i+1}^j\},$$
$$E(C_{3,i}^j) = \{cu_i^j, u_i^j u_{i+1}^j, cu_i^j\}.$$

The upper bound for the parameter d under super (a, d)-C_3-antimagic covering of mW_n.

Theorem 3. *For every $m \geq 2$ and $n \geq 3$, the parameter $d \leq 9$ for a super (a, d)-C_3-antimagic covering of mW_n.*

Proof. Let ξ be a super (a, d)-C_3-antimagic covering of the graph mW_n then minimum possible weight of C_3 is atleast $\sum_{i=1}^{3} i + \sum_{i=1}^{3} (mn + 1 + i) = 3mn + 15$ and the largest weight of C_3 is no more than $\sum_{i=1}^{3} (mn + 2 - i) + \sum_{i=1}^{3} (3mn + 2 - i) = 12mn$. Thus $a + (mn - 1)d \leq 12mn$ and $d \leq \left\lceil 9 - \frac{6}{mn-1} \right\rceil$. For $m \geq 2$ and $n \geq 3$, we get $d \leq 9$. $\square$

Theorem 4. *For every $n \geq 3$ and $m \geq 2$, the graph mW_n admits a super (a, d)-C_3-antimagic total labeling for $d = \{1, 3\}$.*

Proof. Let $f_d : V(mW_n) \cup E(mW_n) \rightarrow \{1, 2, 3, ..., 3mn + 1\}$ is a total labeling define as:

$$f_d(c) = 1,$$
$$f_1(u_i^j) = jn + 2 + i, \qquad \text{for } 0 \leq i \leq n-1,\ 0 \leq j \leq m-1,$$
$$f_1(cu_i^j) = mn + (m - j)n + 1 - i, \qquad \text{for } 0 \leq i \leq n-1,\ 0 \leq j \leq m-1,$$

$$f_1(u_i^j u_{i+1}^j) = 2mn + jn + 2 + i, \qquad \text{for } 0 \le i \le n-1, \ 0 \le j \le m-1,$$

$$f_3(u_i^j) = \begin{cases} mn - 2m + 2 + j, & \text{for } 0 \le j \le m-1, \\ 2m + 1 - j, & \text{for } i = 1, \ 0 \le j \le m-1, \\ 2 + j, & \text{for } i = n-2, \ 0 \le j \le m-1, \\ im + 2 + j, & \text{for } 2 \le i \le n-3, \ 0 \le j \le m-1, \\ (n-1)m + 2 + j, & \text{for } i = n-1, \ 0 \le j \le m-1, \end{cases}$$

$$f_3(u_i^j u_{i+1}^j) = \begin{cases} 3mn + 1 - j, & \text{for } i = n-3, \ 0 \le j \le m-1, \\ 3mn - m + 1 - j, & \text{for } i = n-2, \ 0 \le j \le m-1, \\ 2mn + 2m + 1 - j, & \text{for } i = n-1, \ 0 \le j \le m-1, \\ mn + 2 + j, & \text{for } i = 0, \ 0 \le j \le m-1, \\ 3mn - 3m + 2 + j, & \text{for } i = 1, \ 0 \le j \le m-1, \\ 2mn + (n-1-i)m + 1 - j, & \text{for } 2 \le i \le n-4, \ 0 \le j \le m-1, \end{cases}$$

$$f_3(cu_i^j) = \begin{cases} mn + im + 2 + j, & \text{for } 1 \le i \le n-1, \ 0 \le j \le m-1, \\ 2mn + 2 + j, & \text{for } i = 0, \ 0 \le j \le m-1. \end{cases}$$

For every $0 \le i \le n-1$ and $0 \le j \le m-1$ the weights of $C_{3,i}^j$ are as follows:

$$wt_d(C_{3,i}^j) = f_d(c) + f_d(u_i^j) + f_d(u_{i+1}^j) + f_d(cu_i^j) + f_d(u_i^j u_{i+1}^j + f_d(cu_{i+1}^j),$$

$$wt_d(C_{3,i}^j) = \begin{cases} jn + i + 6mn + 9, & \text{for } d = 1, \\ 5mn + m + 3j + 3im + 10, & \text{for } d = 3. \end{cases}$$

When we put values of i and j from $0 \le i \le n-1$ and $0 \le j \le m-1$ in $wt_d(C_{3,i}^j)$, we get the sets $\{6mn + 9, 6mn + 10, 6mn + 11, ..., 7mn + 8\}$ and $\{5mn + m + 10, 5mn + m + 13, 5mn + m + 16, ..., 8mn + m + 7\}$ form an arithmetic sequence with differences 1 and 3. $\square$

4. Conclusion

In this paper, we construct upper bound for the parameter d and examine the existence of (a, d)-C_8-supermagic labelings for octagonal grid graph O_m^n for all $m \ge 3$, $n \ge 2$ and for $d \in \{2, 4, 6, 8, 10, 12, 14, 16\}$. Moreover we showed that the multiple wheel graph mW_n for $m \ge 2$ and $n \ge 3$ admits (a, d)-C_3-supermagic labeling for $d \in \{1, 3\}$.

Author Contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] Gross, J. L., & Yellen, J. (Eds.). (2003). *Handbook of Graph Theory.* CRC press.

[2] Simanjuntak, R., Bertault, F., & Miller, M. (2000). Two new $(a, d)-$antimagic graph labelings. *In Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms, 11,* 179-189.

[3] Gutiérrez, A., & Lladó, A. (2005). Magic coverings. *Journal of Combinatorial Mathematics and Combinatorial Computing, 55,* 43–46.

[4] Lladó, A., & Moragas, J. (2007). Cycle-magic graphs. *Discrete Mathematics, 307*(23), 2925-2933.

[5] Jeyanthi, P., & Selvagopal, P. (2010). More classes of H-supermagic graphs. *International Journal of Algorithms, Computing and Mathematics, 3*(1), 93-108.

[6] Maryati, T. K., Salman, A. N. M., Baskoro, E. T., Ryan, J., & Miller, M. (2010). On H-supermagic labelings for certain shackles and amalgamations of a connected graph. *Utilitas Mathematica, 83,* 333-342.

Open J. Math. Sci. **2021**, *5*, 27-33

33

[7]   Ngurah, A. A. G., Salman, A. N. M., & Susilowati, L. (2010). H-supermagic labelings of graphs. *Discrete Mathematics, 310*(8), 1293-1300.

[8]   Roswitha, M., & Baskoro, E. T. (2012, May). H-magic covering on some classes of graphs. *In AIP Conference Proceedings, 1450*(1), 135-138. American Institute of Physics.

[9]   Rizvi, S. T. R., Ali, K., & Hussain, M. (2017). Cycle-supermagic labelings of the disjoint union of graphs. *Utliltas Mathematica, 104*, 215-226.

[10]   Inayah, N., Salman, A. N. M., & Simanjuntak, R. (2009). On (a, d)-H-antimagic coverings of graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing, 71*(273), 273-281.

[11]   Rizvi, S. T. R., Ali, K., Iqbal, N., & Gulraze, A. (2018). Super edge-magicness of stars like graphs. *Ars Combinatoria, 136*, 405-417.

[12]   Razzaq, M. A., Rizvi, S. T. R., & Ali, K. (2018). H–groupmagic total labelings of families of fan graphs. *Utilitas Mathematica, 108*, 213-219.

[13]   Semanicová–Fenovcíková, A., Baća, M., Lascsáková, M., Miller, M., & Ryan, J. (2015). Wheels are cycle–antimagic. *Electronic Notes in Discrete Mathematics, 48*, 11-18.

[14]   Inayah, N., Simanjuntak, R., Salman, A. N. M., & Syuhada, K. I. A. (2013). Super $(a, d)-$H-antimagic total labelings for shackles of a connected graph H. *The Australasian Journal of Combinatorics, 57*, 127-138.

[15]   Le Bras, R., Gomes, C. P., & Selman, B. (2013, June). Double-wheel graphs are graceful. *In Twenty-Third International Joint Conference on Artificial Intelligence* (2013), 587–593