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Establishment of Kifilideen coefficient tables for positive and negative powers of n and $-n$ of Kifilideen trinomial theorem and other development based on matrix and standardized methods

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Abstract: The generation of coefficients of terms of positive and negative powers of n and $-n$ of Kifilideen trinomial theorem as the terms are progress is stressful and time-consuming which the same problem is identified with coefficients of terms of binomial theorem of positive and negative powers of n and $-n$. This slows the process of producing the series of any particular trinomial expansion. This study established Kifilideen coefficient tables for positive and negative powers of n and $-n$ of the Kifilideen trinomial theorem and other developments based on matrix and standardized methods. A Kifilideen theorem of matrix transformation of the positive power of n of trinomial expression in which three variables x, y, and z are found in parts of the trinomial expression was originated. The development would ease evaluating the trinomial expression's positive power of n. The Kifilideen coefficient tables are handy and effective in generating the coefficients of terms and series of the Kifilideen expansion of trinomial expression of positive and negative powers of n and $-n$.

Keywords: Coefficients tables; Combination; Kifilideen matrix; Positive and negative powers; Kifilideen expansion.

MSC: 28A20, 60A05.

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1. Introduction

Goss [1] and Aljohani [2] indicate that the Binomial theorem for negative and fraction powers of $-n$ and $\frac{a}{b}$ were developed by Sir Isaac Newton (I642 – 1727) in 1665. Bombelli (1572) gave the coefficients of the binomial expansion of $(a+b)^n$ for $n = 1, 2, 3, 4, \ldots, 7$ and Oughtred (1631) provided them for $n = 1, 2, 3, 4, 5, \ldots, 10$ [3–5]. Blaise Pascal (1623 - 1662), a French mathematician, developed the Pascal triangle, also known as the Yanghui triangle, in 1664 to generate the coefficient of terms of the positive power of binomial theorem [6–10]. Although, from the triangle, there is no indication of how the coefficients are progressing from one term to the other. However, from the binomial theorem expansion, it could be deduced which term owns the coefficient in the Pascal triangle. No publication is available for coefficients of negative power of $-n$ either in triangle form or any other form. This delays the process of generating the series of the binomial theorem involving the negative power of $-n$.

A theorem of Kifilideen matrix transformation of the positive power of n of trinomial expression in which three variables x, y, and z are found in parts of the trinomial expression was originated. The development would ease evaluating the trinomial expression's positive power of n.

Kifilideen (2020) developed the Kifilideen trinomial theorem of the positive power of n for the expansion of the form $[x+y+z]^n$ using the matrix approach, which helps in arranging the terms of the expansion in an orderly and standardized manner and ways [11]. It was observed that the process of generating the coefficient of each term of the expansion of positive and negative powers of n and $-n$ of the trinomial theorem is stressful and time-consuming which the same problem is identified with coefficients of terms of a binomial theorem of powers of n and $-n$ [12,13]. This slows the process of producing the series of any particular trinomial expansion.

Table 1. Pascal coefficient table for terms of series of expansion of positive power of n of Newton's binomial theorem

Positive power of n																			
Terms	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10	n = 11	n = 12	n = 13	n = 14	n = 15	n = 16	n = 17	n = 18	n = 19
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3		1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171
4			1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	816	969
5				1	5	15	35	70	126	210	330	495	715	1001	1365	1820	2380	3060	3876
6					1	6	21	56	126	252	462	792	1287	2002	3003	4368	6188	8568	11628
7						1	7	28	84	210	462	924	1716	3003	5005	8008	12376	18564	27132
8							1	8	36	120	330	792	1716	3432	6435	11440	19448	31824	50388
9								1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582
10									1	10	55	220	715	2002	5005	11440	24310	48620	92378
11										1	11	66	286	1001	3003	8008	19448	43758	92378
12											1	12	78	364	1365	4368	12376	31824	75582
13												1	13	91	455	1820	6188	18564	50388
14													1	14	105	560	2380	8568	27132
15														1	15	120	680	3060	11628
16															1	16	136	816	3876
17																1	17	153	969
18																	1	18	171
19																		1	19
20																			1

The establishment of tables of coefficients of positive and negative powers of binomial and trinomial theorems would help and be handy in generating the coefficient of each term of a series and result in easy expansion. In the Tables, each term and its corresponding coefficient will be indicated and linked together. Also, it would show how the coefficients are progressing from one term to the other. This will remove the stress that is been encountered in the expansion process if the Tables are fully utilized. It can serve as a guide and would also help in easy visualization of patterns and analysis in which the coefficients are progressing for each power of n of the binomial and trinomial theorems. Kifilideen matrix approach had been used to evaluate and compute the power of base of eleven, other bi-digits, and tri-digit numbers [14–18]. This study established Kifilideen coefficient tables for positive and negative powers of n and $-n$ for the Kifilideen trinomial theorem.

2. 2. Materials and Methods

2.1. Pascal coefficient table for terms of series of expansion of positive power of n of Newton's binomial theorem

Table 1 indicates Pascal coefficient table for terms of series of expansion of positive power of n of Newton's binomial theorem. The series of the terms of Newton's binomial theorem for a particular positive power of n is finite.

2.2. Kifilideen coefficient table for negative power of $-n$ of Newton's binomial theorem

Table 2 shows Kifilideen coefficient table for negative power of $-n$ of Newton's binomial theorem. The series of Newton's expansion of negative power of $-n$ of binomial theorem gives infinite series which can be shown in the Table below. The series of the terms of Newton's binomial theorem for a particular negative power of n is infinite.

2.3. Kifilideen power combination table of the terms of the series of Kifilideen expansion of positive power of n of trinomial expression $(x + y + z)$ in a standardized order

Table 3 presents the Kifilideen power combination table of the terms of the series of Kifilideen expansion of positive power of n of trinomial expression $(x + y + z)$ in a standardized order. Each degree of the positive power of n of the Kifilideen expansion of the Kifilideen trinomial theorem has a finite power combination in the series.

2.4. Kifilideen coefficient table for positive power of n of Kifilideen trinomial theorem

Table 4 presents the Kifilideen coefficient table for positive power of n of Kifilideen trinomial theorem based on matrix approach. The series of Kifilideen expansion of positive power of n of trinomial theorem gives finite series which can be shown in the Table below. From the Table 4; the change in colour from blue to red then to blue and so on till the last term for any particular positive power of n indicates the migration

Table 2. The Kifilideen coefficient table for negative power of $-n$ of Newton's binomial theorem

Negative power of $- -$ *n*												
Terms	n = − 1	n = − 2	n = −3	n = − 4	n = − 5	n = − 6	n = − 7	n = − 8	n = − 9	n = − 10	n = − 11	n = − 12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12
3	1	3	6	10	15	21	28	36	45	55	66	78
4	-1	-4	-10	-20	-35	-56	-84	-120	-165	-220	-286	-364
5	1	5	15	35	70	126	210	330	495	715	1001	1365
6	-1	-6	-21	-56	-126	-252	-462	-792	-1287	-2002	-3003	-4368
7	1	7	28	84	210	462	924	1716	3003	5005	8008	12376
8	-1	-8	-36	-120	-330	-792	-1716	-3432	-6435	-11440	-19448	-31824
9	1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582
10	-1	-10	-55	-220	-715	-2002	-5005	-11440	-24310	-48620	-92378	-167960
11	1	11	66	286	1001	3003	8008	19448	43758	92378	184756	352716
12	-1	-12	-78	-364	-1365	-4368	-12376	-31824	-75582	-167960	-352716	-705432
13	1	13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078
14	-1	-14	-105	-560	-2380	-8568	-27132	-77520	-203490	-497420	-1144066	-2496144
15	1	15	120	680	3060	11628	38760	116280	319770	817190	1961256	4457400
16	-1	-16	-136	-816	-3876	-15504	-54264	-170544	-490314	-1307504	-3268760	-7726160
17	1	17	153	969	4845	20349	74613	245157	735471	2042975	5311735	13037895
18	-1	-18	-171	-1140	-5985	-26334	-100947	-346104	-1081575	-3124550	-8436285	-21474180
19	1	19	190	1330	7315	33649	134596	480700	1562275	4686825	13123110	34597290
20	-1	-20	-210	-1540	-8855	-42504	-177100	-657800	-2220075	-6906900	-20030010	-54627300
21	1	21	231	1771	10626	53130	230230	888030	3108105	10015005	30045015	84672315
22	-1	-22	-253	-2024	-12650	-65780	-296010	-1184040	-4292145	-14307150	-44352165	-129024480
23	1	23	276	2300	14950	80730	376740	1560780	5852925	20160075	64512240	193536720
24	-1	-24	-300	-2600	-17550	-98280	-475020	-2035800	-7888725	-28048800	-92561040	-286097760
25	1	25	325	2925	20475	118755	593775	2629575	10518300	38567100	131128140	417225900
26	-1	-26	-351	-3276	-23751	-142506	-736281	-3365856	-13884156	-52451256	-183579396	-600805296
27	1	27	378	3654	27405	169911	906192	4272048	18156204	70607460	254186856	854992152
28	-1	-28	-406	-4060	-31465	-201376	-1107568	-5379616	-23535820	-94143280	-348330136	-1203322288
29	1	29	435	4495	35960	237336	1344904	6724520	30260340	124403620	472733756	1676056044
30	-1	-30	-465	-4960	-40920	-278256	-1623160	-8347680	-38608020	-163011640	-635745396	-2311801440
31	1	31	496	5456	46376	324632	1947792	10295472	48903492	211915132	847660528	3159461968
32	-1	-32	-528	-5984	-52360	-376992	-2324784	-12620256	-61523748	-273438880	-1121099408	-4280561376
33	1	33	561	6545	58905	435897	2760681	15380937	76904685	350343565	1471442973	5752004349
34	-1	-34	-595	-7140	-66045	-501942	-3262623	-18643560	-95548245	-445891810	-1917334783	-7669339132
35	1	35	630	7770	73815	575757	3838380	22481940	118030185	563921995	2481256778	10150595910
36	-1	-36	-666	-8436	-82251	-658008	-4496388	-26978328	-145008513	-708930508	-3190187286	-13340783196
37	1	37	703	9139	91390	749398	5245786	32224114	177232627	886163135	4076350421	17417133617
38	-1	-38	-741	-9880	-101270	-850668	-6096454	-38320568	-215553195	-1101716330	-5178066751	-22595200368
39	1	39	780	10660	111930	962598	7059052	45379620	260932815	1362649145	6540715896	29135916264
40	-1	-40	-820	-11480	-123410	-1086008	-8145060	-53524680	-314457495	-1677106640	-8217822536	-37353738800
41	1	41	861	12341	135751	1221759	9366819	62891499	377348994	2054455634	10272278170	47626016970
42	-1	-42	-903	-13244	-148995	-1370754	-10737573	-73629072	-450978066	-2505433700	-12777711870	-60403728840
43	1	43	946	14190	163185	1533939	12271512	85900584	536878650	3042312350	15820024220	76223753060
44	-1	-44	-990	-15180	-178365	-1712304	-13983816	-99884400	-636763050	-3679075400	-19499099620	-95722852680
45	1	45	1035	16215	194580	1906884	15890700	115775100	752538150	4431613550	23930713170	1.19654E+11
46	-1	-46	-1081	-17296	-211876	-2118760	-18009460	-133784560	-886322710	-5317936260	-29248649430	-1.48902E+11
47	1	47	1128	18424	230300	2349060	20358520	154143080	1040465790	6358402050	35607051480	1.84509E+11
48	-1	-48	-1176	-19600	-249900	-2598960	-22957480	-177100560	-1217566350	-7575968400	-43183019880	-2.27692E+11
49	1	49	1225	20825	270725	2869685	25827165	202927725	1420494075	8996462475	52179482355	2.79872E+11
50	-1	-50	-1275	-22100	-292825	-3162510	-28989675	-231917400	-1652411475	-10648873950	-62828356305	-3.427E+11

Table 3. Kifilideen power combination table for positive power of n of Kifilideen trinomial theorem based on matrix approach

	Positive power of n											
Terms	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$	$n=11$	$n=12$
1	100	200	300	400	500	600	700	800	900	1000	1100	1200
2	010	110	210	310	410	510	610	710	810	910	1010	1110
3	001	020	120	220	320	420	520	620	720	820	920	1020
4		101	030	130	230	330	430	530	630	730	830	930
5		011	201	040	140	240	340	440	540	640	740	840
6		002	111	301	050	150	250	350	450	550	650	750
7			021	211	401	060	160	260	360	460	560	660
8			102	121	311	501	070	170	270	370	470	570
9			012	031	221	411	601	080	180	245	380	480
10			003	202	131	321	511	701	090	190	290	390
11				112	041	231	421	611	801	0100	1100	2100
12				022	302	141	331	521	711	901	0110	1110
13				103	212	051	241	431	621	811	1001	0120
14				013	122	402	151	341	531	721	911	1101
15				004	032	312	061	251	441	631	821	1011
16					203	222	502	161	351	541	731	921
17					113	132	412	071	261	451	641	831
18					023	042	322	602	171	361	551	741
19					104	303	232	512	081	271	461	651
20					014	213	142	422	702	181	371	561
21					005	123	052	332	612	091	281	471
22						033	403	242	522	802	191	381
23						204	313	152	432	712	0101	291
24						114	223	062	342	622	902	1101
25						024	133	503	252	532	812	0111
26						105	043	413	162	442	722	1002
27						015	304	323	072	352	632	912
28						006	214	233	603	262	542	822
29							124	143	513	172	452	732
30							034	053	423	082	362	642
31							205	404	333	703	272	552
32							115	314	243	613	182	462
33							025	224	153	523	092	372
34							106	134	063	433	803	282
35							016	044	504	343	713	192
36							007	305	414	253	623	0102
37								215	324	163	533	903
38								125	234	073	443	813
39								035	144	604	353	723
40								206	054	514	263	633
41								116	405	424	173	543
42								026	315	334	083	453
43								107	225	244	704	363
44								017	135	154	614	273
45								008	045	064	524	183
46									306	505	434	093
47									216	415	344	804
48									126	325	254	714
49									036	235	164	624
50									207	145	074	534
51									117	055	605	444
52									027	406	515	354
53									108	316	425	264
54									018	226	335	174
55									009	136	245	084
56										046	155	705
57										307	065	615
58										217	506	525
59										127	416	435
60										037	326	345
61										208	236	255
62										118	146	165
63										028	056	075
64										109	407	606
65										019	317	516
66										0010	227	426
67											137	336
68											047	246
69											308	156
70											218	066
71											128	507
72											038	417
73											209	327
74											119	237
75											029	147
76											1010	057
77											0110	408
78											0011	318
79												228
80												138
81												048
82												309
83												219
84												129
85												039
86												2010
87												1110
88												0210
89												1011
90												0211
91												0012

from one group to another. Also, from the Table 4, the maximum number of terms generated by any positive power of n can be known. The formula to determine the maximum number of terms generated by any positive power of n of a trinomial theorem is presented in the Kifilideen (2020) on the publication of development of Kifilideen trinomial theorem using matrix approach [11].

2.5. Kifilideen coefficient table for negative power of -n of Kifilideen trinomial theorem based on matrix and standardized approach

Table 5 shows Kifilideen coefficient table for negative power of $-n$ of Kifilideen trinomial theorem based on matrix and standardized approach. The series of Kifilideen expansion of negative power of $-n$ of trinomial theorem gives infinite series which can be shown in the Table 5 below. From the Table 5; the change in colour from blue to red then to blue and so on to infinity for any particular negative power of $-n$ indicates the migration from one group to another. A clear study of Table 5 indicates that the coefficients of the negative power of -1 of Kifilideen trinomial theorem in periodicity and orderly manner can be generated from coefficients of Pascal triangle of binomial theorem of positive power of n. The coefficients of the negative power of -1 of Kifilideen trinomial theorem in orderly manner are 1; -1, -1; 1, 2, 1; -1, -3, -3, -1; 1, 4, 6, 4, 1; -1, $-$ 5, -10, $-$ 10, -5, $-$ 1; 1, 6, 15, 20, 15, 6, 1;...

Also, the study of the Table 5 reveals that the coefficient of the negative power of -2 of Kifilideen trinomial theorem in periodicity and orderly manner can be generated from the coefficients of Pascal triangle of binomial theorem of positive power of n. The coefficients of the negative power of -2 of Kifilideen trinomial theorem in orderly manner are 1 ($\times$1); -1, -1($\times$2); 1, 2, 1 ($\times$3); -1, -3, -3, -1 ($\times$4); 1, 4, 6, 4, 1 ($\times$5); -1, $-$ 5, -10, $-$ 10, -5, $-$ 1 ($\times$6); 1, 6, 15, 20, 15, 6, 1 ($\times$7);... . So, the coefficients of the negative power of -2 of Kifilideen trinomial theorem in orderly manner are 1; -2, -2; 3, 6, 3; -4, -12, -12, -4; 5, 20, 30, 20, 5; -6, $-$ 30, -60, $-$ 60, -30, $-$ 6; 7, 42, 105, 140, 105, 42, 7;... .

2.6. Kifilideen theorem of matrix transformation of positive power of n of trinomial expression

If three variables $x, y$and z are found in each part of trinomial expression of positive power of n such as

$$\left[ux^a y^b z^c + vx^d y^e z^g + wx^h y^m z^p\right]^n \tag{1}$$

and the power combination of any term in the Kifilideen expansion of that kind of positive power of the trinomial expression is set as kif while the value of this term is designated as $qx^r y^s z^t$.

Then, the kifilideen matrix transformation of such positive power of n of the trinomial expression is of the form;

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{bmatrix} a & d & h \\ b & e & m \\ c & g & p \end{bmatrix} \begin{bmatrix} k \\ i \\ f \end{bmatrix} = \begin{bmatrix} r \\ s \\ t \end{bmatrix} \tag{2}$$

Thus $k + i + f = n$ and where u, v, w and q are constants

More so,

$$^{-n}_{kif}C u^k v^i w^f = q \tag{3}$$

3. Results

3.1. Utilization of Kifilideen coefficient table for Kifilideen trinomial theorem based standardized and matrix approach

[i] Expand the following using Kifilideen coefficient table

[a] $[x + y + z]^7$ [b] $\left(\frac{x^2}{y} - yz^3 + xz\right)^4$ [c] $(x + y + z)^{-5}$ [d] $(1 + x + y)^{-3}$

Solution

[a] From Kifilideen coefficient Table 3 for positive power of 7, the coefficienst in ascending order are 1,7,21,35, 35, 21,7,1,7,42,105,140,105,42,7,21,105,210,210,105,21,35,140,210,105,21,35,140,210,140,35,105,105,35,21,42,21, 7,7,1. So the expansion of $[x + y + z]^7$ using Kifilideen trinomial theorem based on standardized and matrix approach, we have:

Table 4. Kifilideen coefficient table for positive power of n of Kifilideen trinomial theorem based on matrix and standardized approach

	Positive power of n											
Terms	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10	n = 11	n = 12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	9	10	11	12
3	1	1	3	6	10	15	21	28	36	45	55	66
4		2	1	4	10	20	35	56	84	120	165	220
5		2	3	1	5	15	35	70	126	210	330	495
6		1	6	4	1	6	21	56	126	252	462	792
7			3	12	5	1	7	28	84	210	462	924
8			3	12	20	6	1	8	36	120	330	792
9			3	4	30	30	7	1	9	45	165	495
10			1	6	20	60	42	8	1	10	55	220
11				12	5	60	105	56	9	1	11	66
12				6	10	30	140	168	72	10	1	12
13				4	30	6	105	280	252	90	11	1
14				4	30	15	42	280	504	360	110	12
15				1	10	60	7	168	630	840	495	132
16					10	90	21	56	504	1260	1320	660
17					20	60	105	8	252	1260	2310	1980
18					10	15	210	28	72	840	2772	3960
19					5	20	210	168	9	360	2310	5544
20					5	60	105	720	36	90	1320	5544
21					1	60	21	560	252	10	495	3960
22						20	35	420	756	45	110	1980
23						15	140	168	1260	360	11	660
24						30	210	28	1260	1260	55	132
25						15	140	56	756	2520	495	12
26						6	35	280	252	3150	1980	66
27						6	35	560	36	2520	4620	660
28						1	105	560	84	1260	6930	2970
29							105	280	504	360	6930	7920
30							35	56	1260	45	4620	13860
31							21	70	1680	120	1980	16632
32							42	280	1260	840	4950	13860
33							21	420	504	2520	55	7920
34							7	280	84	4200	165	2970
35							7	70	126	4200	1320	660
36							1	56	630	2520	4620	66
37								168	1260	840	9240	220
38								168	1260	120	11550	1980
39								56	630	210	9240	7920
40								28	126	1260	4620	18480
41								56	126	3150	1320	27720
42								28	504	4200	165	27720
43								8	756	3150	330	18480
44								8	504	1260	2310	7920
45								1	126	210	6930	1980
46									84	252	11550	220
47									242	1260	11550	495
48									252	2520	6930	3960
49									84	2520	2310	13860
50									36	1260	330	27720
51									72	252	462	34650
52									36	210	2772	27720
53									9	840	6930	13860
54									9	1260	9240	3960
55									1	840	6930	495
56										210	6930	792
57										120	2772	5544
58										360	462	16632
59										360	462	27720
60										120	2310	27720
61										45	4620	16632
62										90	4620	5544
63										45	2310	792
64										10	462	924
65										10	330	5544
66										1	1320	13860
67											1980	18480
68											1320	13860
69											330	5544
70											165	924
71											495	792
72											495	3960
73											165	7920
74											55	7920
75											110	3960
76											55	792
77											11	495
78											11	1980
79											1	2970
80												1980
81												495
82												220
83												660
84												660
85												220
86												66
87												132
88												66
89												12
90												12
91												1

Table 5. Kifilideen coefficient table for negative power of $-n$ of Kifilideen trinomial theorem based on matrix and standardized methods

Negative power of − − *n*												
Terms	n = − 1	n = − 2	n = −3	n = − 4	n = − 5	n = − 6	n = − 7	n = − 8	n = − 9	n = − 10	n = − 11	n = − 12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12
3	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12
4	1	3	6	10	15	21	28	36	45	55	66	78
5	2	6	12	20	30	42	56	72	90	110	132	156
6	1	3	6	10	15	21	28	36	45	55	66	78
7	-1	-4	-10	-20	-35	-56	-84	-120	-165	-220	-286	-364
8	-3	-12	-30	-60	-105	-168	-252	-360	-495	-660	-858	-1092
9	-3	-12	-30	-60	-105	-168	-252	-360	-495	-660	-858	-1092
10	-1	-4	-10	-20	-35	-56	-84	-120	-165	-220	-286	-364
11	1	5	15	35	70	126	210	330	495	715	1001	1365
12	4	20	60	140	280	504	840	1320	1980	2860	4004	5460
13	6	30	90	210	420	756	1260	1980	2970	4290	6006	8190
14	4	20	60	140	280	504	840	1320	1980	2860	4004	5460
15	1	5	15	35	70	126	210	330	495	715	1001	1365
16	-1	-6	-21	-56	-126	-252	-462	-792	-1287	-2002	-3003	-4368
17	-5	-30	-105	-280	-630	-1260	-2310	-3960	-6435	-10010	-15015	-21840
18	-10	-60	-210	-560	-1260	-2520	-4620	-7920	-12870	-20020	-30030	-43680
19	-10	-60	-210	-560	-1260	-2520	-4620	-7920	-12870	-20020	-30030	-43680
20	-5	-30	-105	-280	-630	-1260	-2310	-3960	-6435	-10010	-15015	-21840
21	-1	-6	-21	-56	-126	-252	-462	-792	-1287	-2002	-3003	-4368
22	1	7	28	84	210	462	924	1716	3003	5005	8008	12376
23	6	42	168	504	1260	2772	5544	10296	18018	30030	48048	74256
24	15	105	420	1260	3150	6930	13860	25740	45045	75075	120120	185640
25	20	140	560	1680	4200	9240	18480	34320	60060	100100	160160	247520
26	15	105	420	1260	3150	6930	13860	25740	45045	75075	120120	185640
27	6	42	168	504	1260	2772	5544	10296	18018	30030	48048	74256
28	1	7	28	84	210	462	924	1716	3003	5005	8008	12376
29	-1	-8	-36	-120	-330	-792	-1716	-3432	-6435	-11440	-19448	-31824
30	-7	-56	-252	-840	-2310	-5544	-12012	-24024	-45045	-80080	-136136	-222768
31	-21	-168	-756	-2520	-6930	-16632	-36036	-72072	-135135	-240240	-408408	-668304
32	-35	-280	-1260	-4200	-11550	-27720	-60060	-120120	-225225	-400400	-680680	-1113840
33	-35	-280	-1260	-4200	-11550	-27720	-60060	-120120	-225225	-400400	-680680	-1113840
34	-21	-168	-756	-2520	-6930	-16632	-36036	-72072	-135135	-240240	-408408	-668304
35	-7	-56	-252	-840	-2310	-5544	-12012	-24024	-45045	-80080	-136136	-222768
36	-1	-8	-36	-120	-330	-792	-1716	-3432	-6435	-11440	-19448	-31824
37	1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582
38	8	72	360	1320	3960	10296	24024	51480	102960	194480	350064	604656
39	28	252	1260	4620	13860	36036	84084	180180	360360	680680	1225224	2116296
40	56	504	2520	9240	27720	72072	168168	360360	720720	1361360	2450448	4232592
41	70	630	3150	11550	34650	90090	210210	450450	900900	1701700	3063060	5290740
42	56	504	2520	9240	27720	72072	168168	360360	720720	1361360	2450448	4232592
43	28	252	1260	4620	13860	36036	84084	180180	360360	680680	1225224	2116296
44	8	72	360	1320	3960	10296	24024	51480	102960	194480	350064	604656
45	1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582
46	-1	-10	-55	-220	-715	-2002	-5005	-11440	-24310	-48620	-92378	-167960
47	-9	-90	-495	-1980	-6435	-18018	-45045	-102960	-218790	-437580	-831402	-1511640
48	-36	-360	-1980	-7920	-25740	-72072	-180180	-411840	-875160	-1750320	-3325608	-6046560
49	-84	-840	-4620	-18480	-60060	-168168	-420420	-960960	-2042040	-4084080	-7759752	-14108640
50	-126	-1260	-6930	-27720	-90090	-252252	-630630	-1441440	-3063060	-6126120	-11639628	-21162960
51	-126	-1260	-6930	-27720	-90090	-252252	-630630	-1441440	-3063060	-6126120	-11639628	-21162960
52	-84	-840	-4620	-18480	-60060	-168168	-420420	-960960	-2042040	-4084080	-7759752	-14108640
53	-36	-360	-1980	-7920	-25740	-72072	-180180	-411840	-875160	-1750320	-3325608	-6046560
54	-9	-90	-495	-1980	-6435	-18018	-45045	-102960	-218790	-437580	-831402	-1511640
55	-1	-10	-55	-220	-715	-2002	-5005	-11440	-24310	-48620	-92378	-167960
56	1	11	66	286	1001	3003	8008	19448	43758	92378	184756	352716
57	10	110	660	2860	10010	30030	80080	194480	437580	923780	1847560	3527160
58	45	495	2970	12870	45045	135135	360360	875160	1969110	4157010	8314020	15872220
59	120	1320	7920	34320	120120	360360	960960	2333760	5250960	11085360	22170720	42325920
60	210	2310	13860	60060	210210	630630	1681680	4084080	9189180	19399380	38798760	74070360
61	252	2772	16632	72072	252252	756756	2018016	4900896	11027016	23279256	46558512	88884432
62	210	2310	13860	60060	210210	630630	1681680	4084080	9189180	19399380	38798760	74070360
63	120	1320	7920	34320	120120	360360	960960	2333760	5250960	11085360	22170720	42325920
64	45	495	2970	12870	45045	135135	360360	875160	1969110	4157010	8314020	15872220
65	10	110	660	2860	10010	30030	80080	194480	437580	923780	1847560	3527160
66	1	11	66	286	1001	3003	8008	19448	43758	92378	184756	352716
67	-1	-12	-78	-364	-1365	-4368	-12376	-31824	-75582	-167960	-352716	-705432
68	-11	-132	-858	-4004	-15015	-48048	-136136	-350064	-831402	-1847560	-3879876	-7759752
69	-55	-660	-4290	-20020	-75075	-240240	-680680	-1750320	-4157010	-9237800	-19399380	-38798760
70	-165	-1980	-12870	-60060	-225225	-720720	-2042040	-5250960	-12471030	-27713400	-58198140	-116396280
71	-330	-3960	-25740	-120120	-450450	-1441440	-4084080	-10501920	-24942060	-55426800	-116396280	-232792560
72	-462	-5544	-36036	-168168	-630630	-2018016	-5717712	-14702688	-34918884	-77597520	-162954792	-325909584
73	-462	-462	-5544	-36036	-168168	-630630	-2018016	-5717712	-14702688	-34918884	-77597520	-162954792
74	-330	-3960	-25740	-120120	-450450	-1441440	-4084080	-10501920	-24942060	-55426800	-116396280	-232792560
75	-165	-1980	-12870	-60060	-225225	-720720	-2042040	-5250960	-12471030	-27713400	-58198140	-116396280
76	-55	-660	-4290	-20020	-75075	-240240	-680680	-1750320	-4157010	-9237800	-19399380	-38798760
77	-11	-132	-858	-4004	-15015	-48048	-136136	-350064	-831402	-1847560	-3879876	-7759752
78	-1	-12	-78	-364	-1365	-4368	-12376	-31824	-75582	-167960	-352716	-705432
79	1	13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078
80	12	156	1092	5460	21840	74256	222768	604656	1511640	3527160	7759752	16224936
81	66	858	6006	30030	120120	408408	1225224	3325608	8314020	19399380	42678636	89237148
82	220	2860	20020	100100	400400	1361360	4084080	11085360	27713400	64664600	142262120	297457160
83	495	6435	45045	225225	900900	3063060	9189180	24942060	62355150	145495350	320089770	669278610
84	792	10296	72072	360360	1441440	4900896	14702688	39907296	99768240	232792560	512143632	1070845776
85	924	12012	84084	420420	1681680	5717712	17153136	46558512	116396280	271591320	597500904	1249320072
86	792	10296	72072	360360	1441440	4900896	14702688	39907296	99768240	232792560	512143632	1070845776
87	495	6435	45045	225225	900900	3063060	9189180	24942060	62355150	145495350	320089770	669278610
88	220	2860	20020	100100	400400	1361360	4084080	11085360	27713400	64664600	142262120	297457160
89	66	858	6006	30030	120120	408408	1225224	3325608	8314020	19399380	42678636	89237148
90	12	156	1092	5460	21840	74256	222768	604656	1511640	3527160	7759752	16224936
91	1	13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078

$$[x+y+z]^7 = 1\times x^7\times y^0\times z^0+7\ \times x^6\times y^1\times z^0+21\times x^5\times y^2\times z^0+35\ \times x^4\times y^3\times z^0+35\ \times x^3\times y^4\times z^0$$

$$+21\ \times x^2\times y^5\times z^0+7\ \times x^1\times y^6\times z^0+1\ \times x^0\times y^7\times z^0+7\ \times x^6\times y^0\times z^1+42\ \times x^5\times y^1\times z^1+105$$

$$\times x^4\times y^2\times z^1+140\ \times x^3\times y^3\times z^1+105\ \times x^2\times y^4\times z^1+42\ \times x^1\times y^5\times z^1+7\ \times x^0\times y^6\times z^1$$

$$+21\times x^5\times y^0\times z^2+105\ x^4\times y^1\times z^2+210\ x^3\times y^2\times z^2+210\ x^2\times y^3\times z^2+105\ x^1\times y^4\times z^2+21\ x^0\times y^5\times z^2+35$$

$$\times x^4\times y^0\times z^3+140\ \times x^3\times y^1\times z^3+210\ \times x^2\times y^2\times z^3+140\ \times x^1\times y^3\times z^3$$

$$+\times x^0\times y^4\times z^3+35\ \times x^3\times y^0\times z^4+105\ \times x^2\times y^1\times z^4+105\ \times x^1\times y^2\times z^4+35\ \times x^0\times y^3\times z^4+21\ \times x^2\times y^0\times z^5$$

$$+42\ \times x^1\times y^1\times z^5+21\ \times x^0\times y^2\times z^5+7\ \times x^1\times y^0\times z^6+7\ \times x^0\times y^1\times z^6+1\ \times x^0\times y^0\times z^7 \quad (4)$$

$$[x+y+z]^7 = x^7+7\,x^6y+21\,x^5y^2+35\,x^4y^3+35\,x^3y^4+21\,x^2y^5+7\,xy^6+y^7+7\ \times x^6z+42\,x^5yz+105\,x^4y^2z$$

$$+140\,x^3y^3z+105\,x^2y^4z+42\,xy^5z+7\,y^6z+21x^5z^2+105\,x^4yz^2+210\,x^3y^2z^2+210\,x^2y^3z^2+105\,xy^4z^2+21\,y^5z^2$$
$$+35\,x^4z^3+140\,x^3yz^3+210\,x^2y^2z^3+140\,xy^3z^3+35\,y^4z^3+35\,x^3z^4+105\,x^2y^1z^4+105\,xy^2z^4+35\,y^3z^4+21\,x^2z^5$$
$$+42\,xyz^5+21y^2z^5+7xz^6+7\,yz^6+z^7 \quad (5)$$

[b] From Kifilideen coefficient Table 3 for positive power of 4, the coefficients in ascending order are 1,4,6,4,1,4,12,12,4,6,12,6,4,4,1. So the expansion of $\left(\frac{x^2}{y}-yz^3+xz\right)^4$ using Kifilideen trinomial theorem of positive power of n based on standardized and matrix approach, we have:

$$\left(x^2y^{-1}-yz^3+xz\right)^4 = 1\times\left[x^2y^{-1}\right]^4\left[-yz^3\right]^0[xz]^0+4\left[x^2y^{-1}\right]^3\left[-yz^3\right]^1[xz]^0+6\left[x^2y^{-1}\right]^2\left[-yz^3\right]^2[xz]^0+$$

$$+4\left[x^2y^{-1}\right]^1\left[-yz^3\right]^3[xz]^0+\left[x^2y^{-1}\right]^0\left[-yz^3\right]^4[xz]^0+4\left[x^2y^{-1}\right]^3\left[-yz^3\right]^0[xz]^1+12\left[x^2y^{-1}\right]^2\left[-yz^3\right]^1[xz]^1$$
$$+12\left[x^2y^{-1}\right]^1\left[-yz^3\right]^2[xz]^1+4\left[x^2y^{-1}\right]^0\left[-yz^3\right]^3[xz]^1+6\left[x^2y^{-1}\right]^3\left[-yz^3\right]^0[xz]^2+12\left[x^2y^{-1}\right]^2\left[-yz^3\right]^1[xz]^2$$
$$+6\left[x^2y^{-1}\right]^1\left[-yz^3\right]^2[xz]^2+4\left[x^2y^{-1}\right]^1\left[-yz^3\right]^0[xz]^3+4\left[x^2y^{-1}\right]^0\left[-yz^3\right]^1[xz]^3+\left[x^2y^{-1}\right]^0\left[-yz^3\right]^0[xz]^4 \quad (6)$$

$$\left(x^2y^{-1}-yz^3+xz\right)^4 = x^8y^{-4}-4x^6y^{-2}z^3+6x^4z^6-4x^2y^2z^9+y^4z^{12}+4x^7y^{-3}z-12x^5y^{-1}z^4+12x^3yz^7$$
$$-4xy^3z^{10}+6x^8y^{-3}z^2-12x^4y^{-1}z^5+6x^4yz^8+4x^5y^{-1}z^3-4x^3yz^6+x^4z^4 \quad (7)$$

[c] From Kifilideen coefficient Table 4 for negative power of -5, the coefficients in ascending order are $1,-5,-5,15,30,15,-35,-105,-105,-35,70,280,420,280,70,-126,-630,-1260,-1260,-630,-126,$

210, 1260, 3150, 4200, 3150, 1260, 210, − − − . The expansion gives infinite series. So the expansion of $(x+y+z)^{-5}$ using Kifilideen trinomial theorem of negative power of – n based on standardized and matrix approach, we have:

$$(x+y+z)^{-5} = x^{-5}y^0z^0 - -5x^{-6}y^1z^0 - 5x^{-6}y^0z^1 + 15x^{-7}y^2z^0 + 30x^{-7}y^1z^1 + 15x^{-7}y^0z^1 - 35x^{-8}y^3z^0 - 105x^{-8}y^2z^1 - 105x^{-8}y^1z^2 - 35x^{-8}y^0z^3 + 70x^{-9}y^4z^0 + 280x^{-9}y^3z^1 + 420x^{-9}y^2z^2 + 280x^{-9}y^1z^3 + 70x^{-9}y^0z^4 - 126x^{-10}y^5z^0 - 630x^{-10}y^4z^1 - 1260x^{-10}y^3z^2 - 1260x^{-10}y^2z^3 - 630x^{-10}y^1z^4 - 126x^{-10}y^0z^5 + 210x^{-11}y^6z^0 + 1260x^{-10}y^5z^1 + 3150x^{-10}y^4z^2 + 4200x^{-10}y^3z^3 + 3150x^{-10}y^2z^4 + 1260x^{-10}y^1z^5 + 210x^{-11}y^0z^6$$

[d] From Kifilideen coefficient table 4 for negative power of -3, the coefficients in ascending order are 1, −3, −3, 6, 12, 6, −10, −30, −30, −10, 15, 60, 90, 60, 15, −21, −105, −210, −210, −105, −21, 28, 168, 420, 560, 420, 168, 28, − − −−. The expansion gives infinite series. So the expansion of $(1+y+z)^{-3}$ using Kifilideen trinomial theorem of negative power of $-n$ based on standardized and matrix approach, we have:

$$(1+x+y)^{-3} = 1 - 3x^1y^0 - 3x^0y^1 + 6x^2y^0 + 12x^1y^1 + 6x^0y^2 - 10x^3y^0 - 30x^2y^1 - 30x^1y^2 - 10x^0y^3 + 15x^4y^0 + 60x^3y^1 + 90x^2y^2 + 60x^1y^3 + 15x^0y^4 - 21y^5z^0 - 105y^4z^1 - 210y^3z^2 - 210y^2z^3 - 105y^1z^4 - 21y^0z^5 + 28y^6z^0 + 168y^5z^1 + 420y^4z^2 + 560y^3z^3 + 420y^2z^4 + 168y^1z^5 + 28y^0z^6 + \dots \quad (8)$$

[ii] Expand the trinomial expression $[x+y+z]^{-1}$ using Kifilideen trinomial theorem for negative power of $-n$ based on standardized and matrix approach. Obtain the coefficients of the series using Kifilideen coefficient table. Hence or otherwise generate the series of $\frac{35}{82}$ (Hint: take x = 2, y = $\frac{1}{5}$ and z = $\frac{1}{7}$)

Solution

From Kifilideen coefficient Table 4 for negative power of − 1, the coefficients in ascending order are 1, −1, −1, 1, 2, 1, −1, −3, −3, −1, 1, 4, 6, 4, 1, −1, −5, −10, −10, −5, −1, 1, 6, 15, 20, 15, 6, 1, −7, −21. − 35, −35, ... The expansion gives infinite series. So the expansion of $(1+y+z)^{-3}$ using Kifilideen trinomial theorem of negative power of $-n$ based on standardized and matrix approach, we have:

$$[x+y+z]^{-1} = x^{-1}y^0z^0 - x^{-2}y^1z^0 - x^{-2}y^0z^1 + x^{-3}y^2z^0 + 2x^{-3}y^1z^1 + x^{-3}y^0z^2 - x^{-4}y^3z^0 - 3x^{-4}y^2z^1 - 3x^{-4}y^1z^2 - x^{-4}y^0z^3 + x^{-5}y^4z^0 + 4x^{-5}y^3z^1 + 6x^{-5}y^2z^2 + 4x^{-5}y^1z^3 + x^{-5}y^0z^4 - x^{-6}y^5z^0 - 5x^{-6}y^4z^1 - 10x^{-6}y^3z^2 - 10x^{-6}y^2z^3 - 5x^{-6}y^1z^4 - x^{-6}y^0z^5 + x^{-7}y^6z^0 + 6x^{-7}y^5z^1 + 15x^{-7}y^4z^2 + 20x^{-7}y^3z^3 + 15x^{-7}y^2z^3 + 6x^{-7}y^1z^5 + x^{-7}y^0z^6 - x^{-8}y^7z^0 - 7x^{-8}y^6z^1 - 21x^{-8}y^5z^2 - 35x^{-8}y^4z^3 - 35x^{-8}y^3z^3 - 21x^{-8}y^2z^5 - 7x^{-8}y^1z^6 - x^{-8}y^0z^7 + \dots \quad (9)$$

$$[x+y+z]^{-1} = x^{-1} - x^{-2}y - x^{-2}z + x^{-3}y^2 + 2x^{-3}yz + x^{-3}z^2 - x^{-4}y^3 - 3x^{-4}y^2z - 3x^{-4}yz^2 - x^{-4}z^3 + x^{-5}y^4 + 4x^{-5}y^3z + 6x^{-5}y^2z^2 + 4x^{-5}yz^3 + x^{-5}z^4 - x^{-6}y^5 - 5x^{-6}y^4z - 10x^{-6}y^3z^2 - 10x^{-6}y^2z^3 - 5x^{-6}z^4 - x^{-6}z^5 + 6x^{-7}y^5z^1 + 15x^{-7}y^4z^2 + 20x^{-7}y^3z^3 + 15x^{-7}y^2z^3 + 6x^{-7}yz^5 + x^{-7}z^6 - x^{-8}y^7 - 7x^{-8}y^6z^1 - 21x^{-8}y^5z^2 - 35x^{-8}y^4z^3 - 35x^{-8}y^3z^3 - 21x^{-8}y^2z^5 - 7x^{-8}yz^6 - x^{-8}z^7 + \dots \quad (10)$$

[b] $\frac{35}{82} = \left[2 + \frac{1}{5} + \frac{1}{7}\right]^{-1}$

Using the Kifilideen expansion above where x = 2, y = $\frac{1}{5}$ and z = $\frac{1}{7}$, we have:

$$\frac{35}{82} = \left[2 + \frac{1}{5} + \frac{1}{7}\right]^{-1} = \frac{1}{2} - \frac{1}{4\times 5} - \frac{1}{4\times 7} + \frac{1}{8\times 25} + \frac{2}{8\times 5\times 7} + \frac{1}{8\times 49} - \frac{1}{16\times 125} - \frac{3}{16\times 25\times 7} - \frac{3}{16\times 5\times 49}$$

$$-\frac{1}{16\times 343}+\frac{1}{32\times 625}+\frac{4}{32\times 125\times 7}+\frac{6}{32\times 25\times 49}+\frac{4}{32\times 5\times 343}+\frac{1}{32\times 2401}-\frac{1}{64\times 3125}-\frac{5}{64\times 625\times 7}$$

$$-\frac{10}{64\times 125\times 49}-\frac{10}{64\times 25\times 343}-\frac{5}{64\times 5\times 2401}-\frac{1}{64\times 16807}+\frac{1}{128\times 15625}+\frac{6}{128\times 3125\times 7}+\frac{15}{128\times 625\times 49}$$

$$+\frac{20}{128\times 125\times 343}+\frac{15}{128\times 25\times 2401}+\frac{6}{128\times 5\times 16807}+\frac{1}{128\times 117649}-\frac{1}{256\times 78125}-\frac{7}{256\times 15625\times 7}$$

$$-\frac{21}{256\times 3125\times 49}-\frac{35}{256\times 625\times 343}-\frac{35}{256\times 125\times 2401}-\frac{21}{256\times 25\times 16807}-\frac{7}{256\times 5\times 117649}-\frac{1}{256\times 823543}\cdots \quad (11)$$

The evaluation of the above series gives 0.426829 to 6 decimal places. Also, using calculator $\frac{35}{82}$ gives 0.426829 to 6 decimal places. This indicates that the negative power of -1 of Kifilideen trinomial theorem and coefficient of negative power of -1 from the Kifilideen Coefficient table are valid.

[iii] Expand the trinomial expression $[x+y+z]^3$ using Kifilideen trinomial theorem for positive power of n based on standardized and matrix approach. Obtain the coefficients of the series using Kifilideen coefficient table. Hence or otherwise generate the series of $[3.74]^3$ and evaluate its value (Hint: take x = 3, y = 0.7 or $\frac{7}{10}$ and z = 0.04 or $\frac{4}{100}$).

Solution

[a] From Kifilideen coefficient Table 3 for positive power of 3, the coefficients in ascending order are 1, 3, 3, 1, 3, 6, 3, 3, 3, 1. So the expansion of $[x+y+z]^3$ using Kifilideen trinomial theorem based on standardized and matrix approach, we have:

$$[x+y+z]^3 = x^3y^0z^0+3x^2y^1z^0+3x^1y^2z^0+x^0y^3z^0+3x^2y^0z^1+6x^1y^1z^1+3x^0y^2z^1+3x^1y^0z^2+3x^0y^1z^2 +x^0y^0z^3 \quad (12)$$

$$[x+y+z]^3 = x^3+3x^2y+3xy^2+y^3+3x^2z+6xyz+3y^2z+3xz^2+3yz^2+z^3 \quad (13)$$

[b] $[3.74]^3 = \left[3+\frac{7}{10}+\frac{4}{100}\right]^3$

Using the kif expansion above where x = 3, y = $\frac{7}{10}$ and z = $\frac{4}{100}$, we have

$$[3.74]^3 = \left[3+\frac{7}{10}+\frac{4}{100}\right]^3 = 27+\frac{189}{10}+\frac{441}{100}+\frac{343}{1000}+\frac{108}{100}+\frac{504}{1000}+\frac{588}{10000}+\frac{144}{10000}+\frac{336}{100000}+\frac{64}{1000000} \quad (14)$$

$$[3.74]^3 = 27+18.9+4.41+0.343+1.08+0.504+0.0588+0.0144+0.00336+0.000064 \quad (15)$$

$$[3.74]^3 = 52.313624 \quad (16)$$

From calculator, the value of $[3.74]^3$ is also 52.313624. This indicates that the positive power of 3 of Kifilideen trinomial theorem and coefficient of positive power of 3 of trinomial expression from the Kifilideen Coefficient table are valid.

[iv] Determine the power combination of the Kifilideen expansion of negative power of -3 in the row 13 and column 10 of the Kif matrix of the Kifilideen expansion $[x+y+z]^{-3}$. Hence or otherwise determine the term the generate the power combination and using Kifilideen coefficient table of negative power of $-n$ of trinomial theorem determine the coefficient of the power combination.

Solution

[a] From the question, $n=-3,\ r=13\ and\ c=10$

Using Kifilideen General Row Column Matrix formula for negative power of $-n$ [12],

$$CP_{rc} = n00 - 110\,(r-1) + 9(r-c) \tag{17}$$

$$CP_{rc} = -300 - 110\,(13-1) + 9(13-10) \tag{18}$$

$$CP_{rc} = -1593 \tag{19}$$

[b] From Kifilideen General Term formula for negative power of $-n$ [12],

$$t = \frac{[n-k]^2 + [n-k] + 2f + 2}{2}$$

Where,
$t-$ the required tth term
$k-$ the first component part of the power combination
$i-$ the second component part of the power combination
$f-$ the third component part of the power combination
$n-$the negative power of – n of the trinomial expression
From, $CP_{rc} = -1593, k = -15,\ i = 9\ and\ f = 3$

$$t = \frac{[-3 - -15]^2 + [-3 - -15] + 2 \times 3 + 2}{2}$$

$t = 82^{th}$ term
Using the Kifilideen Coefficient table of negative power of -3,
The coefficient of $CP_{rc} = -1593$ of term 82^{th} term is + 20020.

3.2. Demonstration on the implementation of Kifilideen theorem of matrix transformation of positive power of n of trinomial expression

Given that a term in the Kifilideen expansion of $\left[\frac{y^3}{4x^2z^{\ 5}} - \frac{x^7z^{\ 4}}{2y} + \frac{ey^6z^{\ 8}}{x}\right]^n$ is $\frac{-1575}{2048}x^{\ 26}y^{13}z^{\ 8}$ where e is a constant value. Using Kifilideen matrix transformation method of positive power of n of trinomial expression. Find

[i] the power combination
[ii] the degree of the positive power of the trinomial expression
[iii] the tth term
[iv] the value of e.

Solution

[i] Trinomial expression: $\left[4^{-1}x^{-2}y^3z^{-5} - 2^{-1}x^7y^{-1}z^4 + ex^{-1}y^6z^8\right]^n$
Power combination to be obtained: $k \quad i \quad f$
t^{th} term of the power combination: $\frac{-1575}{2048}x^{\ 26}y^{13}z^{\ 8}$
Using the kifilideen matrix transformation method, so

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{bmatrix} -2 & 7 & -1 \\ 3 & -1 & 6 \\ -5 & 4 & 8 \end{bmatrix} \begin{bmatrix} k \\ i \\ f \end{bmatrix} = \begin{bmatrix} 26 \\ 13 \\ 8 \end{bmatrix} \tag{20}$$

Also, $k + i + f = n$
Using Crammer's rule, so

$$k = \frac{\Delta k}{\Delta} = \frac{\begin{vmatrix} 26 & 7 & -1 \\ 13 & -1 & 6 \\ 8 & 4 & 8 \end{vmatrix}}{\begin{vmatrix} -2 & 7 & -1 \\ 3 & -1 & 6 \\ -5 & 4 & 8 \end{vmatrix}}$$

$$k = \frac{-1284}{-321} \tag{21}$$

$$k = 4 \tag{22}$$

$$i = \frac{\Delta i}{\Delta} = \frac{\begin{vmatrix} -2 & 26 & -1 \\ 3 & 13 & 6 \\ -5 & 8 & 8 \end{vmatrix}}{\begin{vmatrix} -2 & 7 & -1 \\ 3 & -1 & 6 \\ -5 & 4 & 8 \end{vmatrix}}$$

$$i = \frac{-1605}{-321} \tag{23}$$

$$i = 5 \tag{24}$$

$$f = \frac{\Delta f}{\Delta} = \frac{\begin{vmatrix} -2 & 7 & 26 \\ 3 & -1 & 13 \\ -5 & 4 & 8 \end{vmatrix}}{\begin{vmatrix} -2 & 7 & -1 \\ 3 & -1 & 6 \\ -5 & 4 & 8 \end{vmatrix}}$$

$$f = \frac{-321}{-321} \tag{25}$$

$$f = 1 \tag{26}$$

So, the power combination $= kif = 451$

[ii] the positive power of n of the trinomial expression $= n = k + i + f = 4 + 5 + 1$

$$n = 10 \tag{27}$$

[iii] $C_p = kif = 451$

$$k = 4,\ i = 5\ and\ f = 1 \tag{28}$$

$$n = k + i + f = 4 + 5 + 1 = 10 \tag{29}$$

From Kifilideen general term formula [19],

$$t = \frac{-f^2 + 2fn + 3f + 2i + 2}{2}$$

Where,
C_p- the given power combination
$k, i, f-$ the component parts of the power combination
$f-$the third component part of the given power combination
$i-$the second component part of the given power combination
$n-$the degree of the positive power of the trinomial expression
$t-$ the term of the given power combination to be determined

$$t = \frac{-[1]^2 + 2 \times 1 \times 10 + 3 \times 1 + 2 \times 5 + 2}{2}$$

$t = 17^{th}$ term

OR

Using Kifilideen general power combination formula [11,20,21],

$$C_p = kif = 451 \tag{30}$$

$$k = 4,\ i = 5\ and\ f = 1 \tag{31}$$

$$n = k + i + f = 4 + 5 + 1 = 10 \tag{32}$$

$$a = f = 1 \tag{33}$$

$$m = \frac{a}{2}\left[2n - a - 1\right] = \frac{1}{2}\left[2 \times 10 - 1 - 1\right] = 9 \tag{34}$$

$$C_p = kif = -90t + 81a + 90m + n90$$

Where,

C_P − Power combination, kif

t- nth term of the kifilideen trinomial theorem

a- the power of the third digit, f of the power combination or the value of the third digit of the column or group the term fall into

a and m - are constant values for a particular group or column of the matrix

n- the power of the trinomial expression

k, i and f −− the first, second and third component part of the power combination

$451 = kif = 90t + 81 \times 1 + 90 \times 9 + 1090$

$t = 17^{th}$ term

OR

Using alternate Kifilideen general power combination formula [19],

From the question, $C_p = kif = 451$

$k = 4,\ i = 5,\ f = 1$ and $n = 10$

$C_p = kif = 90t - 45\, f^2 + 36f + 90\, fn + n90$

$451 = -\ 90t - 45\,[1]^2 + 36 \times 1 + 90 \times 1 \times 10 + 1090$

$t = 17\ ^{th}$ term

[iv] From the question, $u\ =\ 4,\ v\ =\ 1,\ w = e\ and\ q = \frac{-1575}{2048}$

Using Kifilideen matrix transformation method,

$$^{n}_{kif}Cu^k v^i w^f = q \tag{35}$$

$$^{10}_{451}C[4^{-1}]^4[-2^{-1}]^5[e]^1 = \frac{-1575}{2048} \tag{36}$$

From the Kifilideen coefficient table, the coefficient of the 17^{th} term on the n = 10 column is 1260

$$1260 \times \frac{1}{256} \times -\frac{1}{32} \times e = \frac{-1575}{2048} \tag{37}$$

$$e = 5 \tag{38}$$

4. Conclusion

Kifilideen theorem of matrix transformation of positive power of n of trinomial expression in which three variables x, y and z are found in parts of the trinomial expression was established. The research work established Kifilideen coefficient tables for positive and negative powers of n and $-n$ of Kifilideen trinomial theorem base on standardized and matrix approach. The development of theorem of matrix transformation of positive power of n of trinomial expression would make the process of evaluating such positive power of n of the trinomial expression easy. The inaugurated tables had been fully utilized to generate series of expansion for positive and negative power of binomial and trinomial expressions. The Kifilideen coefficient tables are handy and effective in generating the coefficients of terms and series of the Kifilideen expansion of trinomial expression of powers of n and $-n$.

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