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Exact solutions of paraxial wave dynamical model with Kerr law non-linearity using analytical techniques

Naeem Ullah

Department of Mathematics, University of Management and Technology, Lahore, Pakistan; naeemullahjamali@gmail.com

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Abstract: Using the Kudryashov and Tanh methods, we have obtained novel exact solutions for the Paraxial Wave Dynamical Equation with Kerr law, including various types of wave solutions. These distinct types of wave solutions have important applications in physics and engineering, and their physical characteristics are well defined. These outcomes are a substantial innovation in the study of water waves in mathematical physics and engineering phenomena. The results we have acquired demonstrate the power and effectiveness of the present techniques.

Keywords: Kudryashov method; Optical solitons; Paraxial wave dynamical model with Kerr law non-linearity; Tanh method.

MSC: 35E05; 35C08; 35Q51; 37L50; 37J25; 33F05.

1. Introduction

Nonlinear phenomena are an essential area of research that arises in several branches of engineering and physical sciences, including plasma, solid-state physics, optical fibers, chemical kinetics, biology, and fluid mechanics. Nonlinear evolution equations (NLEEs) often play a crucial role in the mathematical representation of these phenomena. Obtaining the solutions of NLEEs can aid in understanding the dynamics of these phenomena. Exact traveling wave solutions (TWSs) of NLEEs have become increasingly important tools in physical phenomena.

Various methods have been used to obtain TWSs for NLEEs in previous works, such as the new extended direct algebraic method [1–3], the first integral method [4,5], the generalized Kudryashov method [6,7], the new extended hyperbolic function method [8], the undetermined coefficient method, and modified mapping method [9], the extended simple equation methods [10], the Jacobi elliptic functions method [11,12], Kudryashov's methods [13,14], the generalized tanh method (GTM) [15], the exp-function method [16], the auxiliary equation method and Sine-Cosine method [17]], the mapping method [18], the Exp $(-f(e))$-expansion method [19], the modified simple equation method [20], the generalized (G'/G) expansion method [21], the modified Khater method [22,23], the extended (G'/G)-expansion method [24,25], the homotopy perturbation double Sumudu transform method [26], the tan method, and the tanh method [27], the fractional extended Fan sub-equation method [28], and the modified auxiliary equation method [33,34], and the sub-equation method [29,30].

In this paper, we focus on constructing novel exact solutions of the Paraxial Wave Dynamical Equation with Kerr law using the Kudryashov and Tanh methods. Our solutions include multiple types of wave solutions, which have important applications in physics and engineering. The physical characteristics of these solutions are well-defined, and the results demonstrate the power and effectiveness of the present techniques.

The paper is structured as follows: §2 presents the governing equation, while §3 discusses the proposed analysis method. In §4, we apply the Kudryashov method, while §5 presents the analysis and application of the tanh method. Finally, we provide the conclusion of this paper in §6.

2. Governing model

Using Kerr media, the dimensionless time-dependent Paraxial wave equation (PWE) [31] through limiting the diffraction into one-dim in the existence of GVD is given as,

$$\iota \frac{\partial E}{\partial Z} + \frac{\alpha}{2}\frac{\partial^2 E}{\partial t^2} + \frac{\beta}{2}\frac{\partial^2 E}{\partial y^2} + \gamma |E|^2 E = 0\,, \tag{1}$$

here, E represents the addition of complex waves, the $t,\ y,$ and z represent temporal, spatial and longitudinal promulgation veriables, correspondingly. In Eq. (1), $\beta,\ \gamma,\ and\ \alpha$ are real numbers and denote the effects of diffraction, non-linearity of Kerr and dispersal, similarly. Eq. (1) convert into hyperbolic NLSE if $\alpha\beta < 0$ and its convert into elliptic NLSE [32] if $\alpha\beta > 0$. Clearly, this model can also describe (2+1)- dim dynamic of spatial in cubic-Kerr media, disregarding GVD, in this sense$y,\ t$ represent the coordinates of temporal and spatial transverse and z denotes the longitudinal coordinate. Moreover $\alpha = \beta\ > 0$,

$$E(y,z,t) = Q(\eta)\ \ e^{\iota Y}, \tag{2}$$

where

$$\eta = k_1 y + k_2 z + wt,\ \ Y = \mu_1 y + \mu_2 z + \tau t + \theta. \tag{3}$$

Using (2) and (3) into (1) and classifying into parts, we get,

$$(\alpha w^2 + \beta k_1^2)Q''(\eta) - (\alpha\tau^2 + \beta\mu_1^2 + 2\mu_2)Q(\eta) - 2\gamma Q(\eta)^3 = 0. \tag{4}$$

$$(2\alpha\tau w + 2\beta k_1\mu_1 + 2k_2)Q'(\eta) = 0. \tag{5}$$

From (5) $Q'(\eta) \neq 0$, so

$$k_2 = -\alpha\tau w - \beta k_1 \mu_1. \tag{6}$$

3. Analysis of the Kudryashov method [13,14]

Suppose we have PDE as follows

$$\Omega(p,\ p_t,\ p_y,\ p_{tt},\ p_{zy},\ p_{zt}, ...) =\ 0, \tag{7}$$

where $p = p(y,z,t)$ is a function. Consider the next wave transformations

$$p(y,z,t) = W(\eta), \qquad \eta = k_1 y + k_2 z + wt. \tag{8}$$

Using (8) we get the following ODE

$$H(W, W', W'', ...) = 0. \tag{9}$$

Consider solutions (9) has the following solution

$$W(\eta) = \sum_{j=0}^{N} f_j (Q(\eta))^j\,, \tag{10}$$

here, f_j are constants and

$$Q(\eta) = \frac{1}{1 + de^{\eta}}\,, \tag{11}$$

that satisfies nonlinear differential equation given, as

$$\frac{dQ}{d\eta} = Q(\eta)(Q(\eta) - 1). \tag{12}$$

Inserting Eq. (12) into Eq. (10), then we found a set of equations by comparing the coefficients of $Q(\eta)$ to zero.

4. Application Kudryashov method

By KM, consider Eq. (4) has the solution, as

$$W(\eta) = f_0 + f_1 Q(\eta), \tag{13}$$

where f_0 and f_1 are constants. Substituting Eq. (13) into Eq. (3), corresponding the coefficients of $Q(\eta)$ to zero, gets a set of equations. On solving the system, the f_j, $j = 0, 1, 2, 3$ are achieved novel sets of solution for Eq. (1).

Case 1

$$f_0 = \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}}, \; f_1 = -\frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}, \; \mu_2 = \frac{1}{4}(-2\alpha\tau^2 - \alpha w^2 - \beta k_1^2 - 2\beta\mu_1^2). \tag{14}$$

Using (13), we got the following solution of (3)

$$W_1(\xi) = \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}} - \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}(1 + de^{(k_1 y + k_2 z + wt)})}. \tag{15}$$

The exact solution for (1) is given by

$$p_1(y, z, t) = \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}} - \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}(1 + de^{(k_1 y + k_2 z + wt)})} \times e^{\iota}(\mu_1 y + \mu_2 z + \tau t + \theta). \tag{16}$$

Case 2

$$f_0 = -\frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}}, \; f_1 = \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}, \; \mu_2 = \frac{1}{4}(-2\alpha\tau^2 - \alpha w^2 - \beta k_1^2 - 2\beta\mu_1^2). \tag{17}$$

Using (13), we got the following solution of (3)

$$W_2(\xi) = -\frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}} + \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}(1 + de^{(k_1 y + k_2 z + wt)})} \tag{18}$$

The exact solution for (1) is given by

$$p_2(y, z, t) = -\frac{\sqrt{\alpha w^2 - \beta k_1^2}}{2\sqrt{\gamma}} + \frac{\sqrt{\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}(1 + de^{(k_1 y + k_2 z + wt)})} \times e^{\iota}(\mu_1 y + \mu_2 z + \tau t + \theta). \tag{19}$$

5. Description of the generalized tanh method [15]

Let PDE as given in (1) with the wave transformation in (2) and (3) using wave transformation ODE is obtained as in (4). We assume that (4) has a solution, as

$$p(y, z, t) = W(\eta) = \sum_{j=0}^{N} f_j Q^j(\eta), \tag{20}$$

here, $f_j(j = (1, 2, 3, ...N))$ are constants and $Q(\eta)$,

$$\frac{dQ}{d\eta} = h + Q^2(\eta)\,. \tag{21}$$

By using balancing rule in (4) the value of N is found. Replacing (20) into (4) with (21), gives a set of equations for $f_j(j = (0, 1, 2, 3, ...N))$. On solving this set, we yield set of solutions that admits (4), as follows

Consider the solutions of (21) are, as

Case 1

If $h < 0$, then

$$Q(\eta) = -\sqrt{-h}\ \ tanh(\sqrt{-h}\eta), \tag{22}$$

and

$$Q(\eta) = -\sqrt{-h}\ \ coth(\sqrt{-h}\eta). \tag{23}$$

5.1. Case 2

If $h = 0$, then

$$Q(\eta) = -\frac{1}{\eta}. \tag{24}$$

Case 3

If $h > 0$, then

$$Q(\eta) = \sqrt{h}\ \ tan(\sqrt{h}\eta), \tag{25}$$

and

$$Q(\eta) = -\sqrt{h}\ \ cot(\sqrt{h}\eta). \tag{26}$$

5.2. Application of Tanh method

According the balancing rule in (4), we get $N = 1$, (20) reduces to

$$W(\xi) = f_0 + f_1 Q(\eta)\,. \tag{27}$$

Putting (27) and (21) in (4) produces a polynomial in form of $Q(\eta)$. We get a system on making a comparison of the coefficients of $Q(\eta)$ to zero, after solving it we get solutions as

Set 1

$f_0 = 0,\ \ f_1 = \frac{\sqrt{-\alpha w^2-\beta k_1^2}}{\gamma}$, $\mu_2 = -\frac{\alpha\tau^2}{2} + h\alpha w^2 + h\beta k_1^2 - \frac{\beta\mu_1^2}{2}$. If $h < 0$, then

$$p_1(y,z,t) = (-\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{-h}\ \ tanh(\sqrt{-h}\eta)) \times e^{\iota(\mu_1 y+\mu_2 z+\tau t+\theta)}, \tag{28}$$

or

$$p_2(y,z,t) = (-\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{-h}\ \ coth(\sqrt{-h}\eta)) \times e^{\iota(\mu_1 y+\mu_2 z+\tau t+\theta)}. \tag{29}$$

If $h = 0$, then

$$p_3(y,z,t) = \frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}(-\frac{1}{\eta}) \times e^{\iota(\mu_1 y+\mu_2 z+\tau t+\theta)}. \tag{30}$$

If $h > 0$, then

$$p_4(y,z,t) = (\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{h}\ \ tan(\sqrt{h}\eta)) \times e^{\iota(\mu_1 y+\mu_2 z+\tau t+\theta)}, \tag{31}$$

or

$$p_5(y,z,t) = (-\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\gamma}\sqrt{h}\ \ cot(\sqrt{h}\eta)) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}. \tag{32}$$

Set 2

$f_0 = 0,\ \ f_1 = -\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}},\ \mu_2 = -\frac{\alpha\tau^2}{2} + h\alpha w^2 + d\beta k_1^2 - \frac{\beta\mu_1^2}{2}$. If $h < 0$, then

$$p_6(y,z,t) = (\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{-h}\ \ tanh(\sqrt{-h}\eta)) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}, \tag{33}$$

or

$$p_7(y,z,t) = (\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{-h}\ \ coth(\sqrt{-h}\eta)) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}. \tag{34}$$

If $h = 0$, then

$$p_8(y,z,t) = -\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}(-\frac{1}{\eta}) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}. \tag{35}$$

If $h > 0$, then

$$p_9(y,z,t) = (-\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{h}\ \ tan(\sqrt{h}\eta)) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}, \tag{36}$$

or

$$p_{10}(y,z,t) = (\frac{\sqrt{-\alpha w^2 - \beta k_1^2}}{\sqrt{\gamma}}\sqrt{h}\ \ cot(\sqrt{h}\eta)) \times e^{\iota(\mu_1 y + \mu_2 z + \tau t + \theta)}. \tag{37}$$

6. Results and discussions

The results of this paper will be valuable for researchers to study the most noticeable applications of the paraxial dynamical model with Kerr law non-linearity in optic fibers. Figures 1-5 reveals the surfaces of the solution acquired for 3-D and 2-D plots, with a selection of suitable parameters for the paraxial dynamical model with Kerr law non-linearity. Likewise, 3D plots provide us to model and exhibit correct physical behavior. Through this study, we consider the optical soliton solutions to the nonlinear paraxial dynamical model with Kerr law non-linearity using the Kudryashov and tanh methods. The authors proposed different analytic approaches in the newly issued article and reported some fascinating findings. The author can understand from all the graphs that the proposed methods are very effective and more specific in assessing the equation under consideration. Figure 1 indicates the solution given by (16), which is dark. Figures 2 and 3 indicate the solutions given by (28) and (29), which are dark and singular, respectively. Figures 4 and 5 are the graphical representations of the solutions given by (36) and (37), which are periodic singular solitons.

7. Conclusion

In this work, we have successfully obtained exact solutions of the paraxial dynamical model with Kerr law non-linearity using the Kudryashov and Tanh methods. These results are of great significance in the study of physical phenomena such as optics and optical fibers, as they can help to better understand the behavior of non-linear optic systems and provide a foundation for future research.

Both the Kudryashov and Tanh methods employed in this study have demonstrated their consistency, efficiency, and effectiveness in solving non-linear PDEs. These methods can be utilized to develop new exact solitons and deepen our understanding of complex physical systems.

The exact solutions obtained in this study have the potential to be applied in a wide range of physical phenomena beyond optics and optical fibers. The Kudryashov and Tanh methods can be adapted and employed in other fields of research, such as fluid mechanics, plasma physics, and quantum mechanics, where non-linear PDEs are frequently encountered.

Overall, the results of this work demonstrate the power and applicability of the Kudryashov and Tanh methods in solving non-linear PDEs and provide a foundation for future research in various fields of physics.

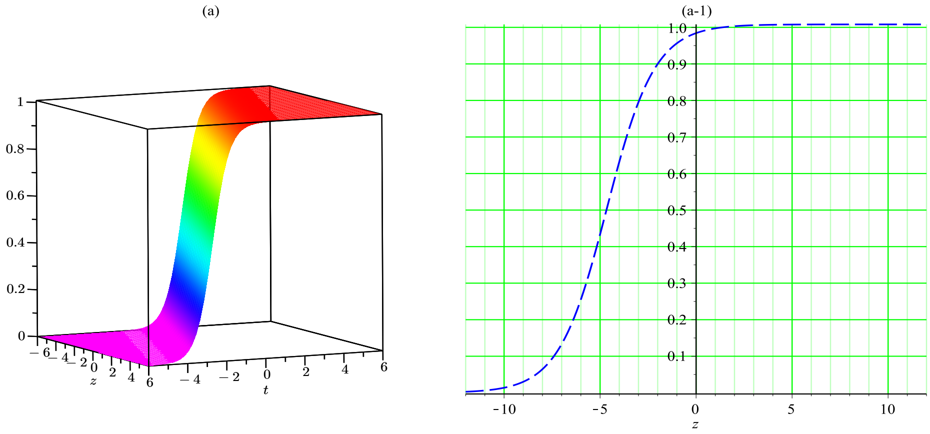

Figure 1. 3-d plot of (16) (left) and (a-1) 2-d plot of (16) with $t = 1$ (right)

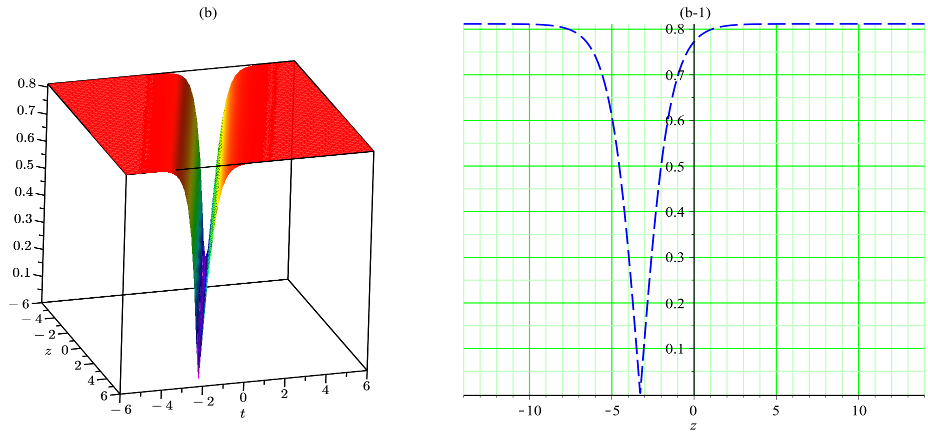

Figure 2. 3-plot of (28) (left) and (b-1) 2-plot of (28) with $t = 1$ (right)

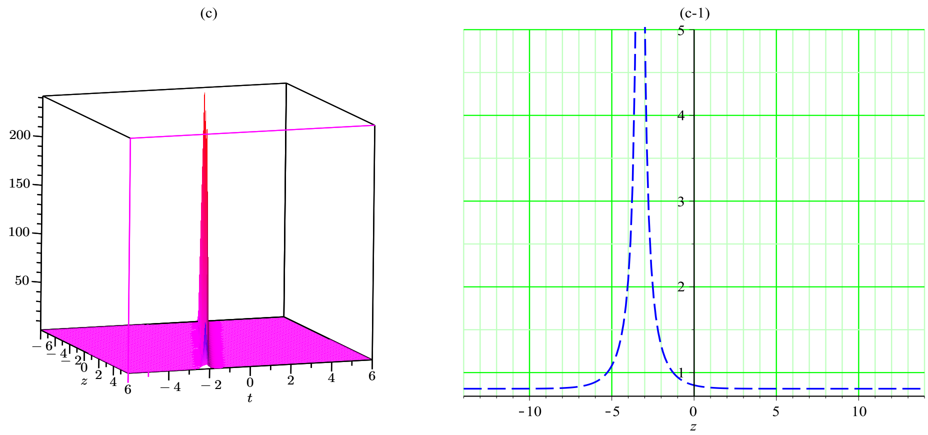

Figure 3. 3-d plot of (29) (left) and (c-1) 2-d plot of (29) with $t = 1$ (right)

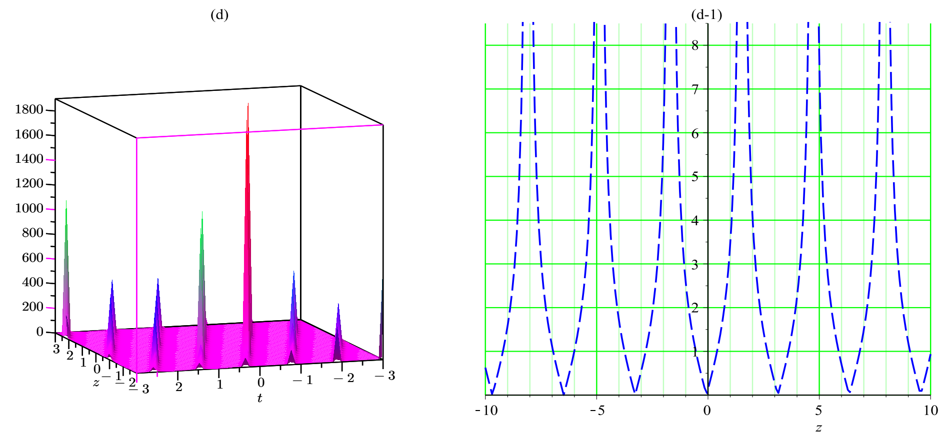

Figure 4. 3-d graph of (36) (left) and (d-1) 2-d plot of (36) with $t = 1$ (right)

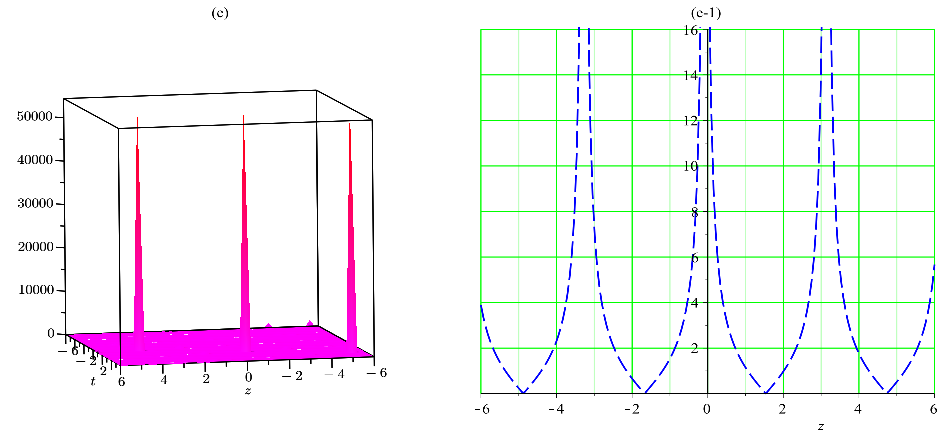

Figure 5. 3-d graph of (37) (left) and (e-1) 2-d plot of (37) with $t = 1$ (right)

Conflicts of Interest: "The author declares no conflict of interest."

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