

## Article

# On model of spatio-temporal distribution of blood in a cardiovascular system

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**Abstract:** In this paper we consider a model for estimation of spatio-temporal distribution of blood in a cardiovascular system. We introduce an approach for analyzing the considered model. We consider a possibility of changing the rate of blood transport.

**Keywords:** blood transport, cardiovascular system, prognosis of process, analytical approach for analysis

## 1. Introduction

The cardiovascular system ensures the transport of blood and all necessary substances to organs and the removal of metabolic products [1–5]. Prognosis of morphological and functional features of the cardiovascular system can prevent or slow down the development of serious diseases that lead to the development of complications in other organs and tissues. Diseases such as arterial hypertension, atherosclerosis, coronary heart disease are widespread and are the most common causes of death due to complications. In this paper we consider a model of blood redistribution between different reservoirs of the cardiovascular system. A model was introduced for assessing the spatiotemporal distribution of blood in the cardiovascular system. We also introduced an analytical approach for analysis of the considered processes.

## 2. Method and results of solution

In this section we consider the following model of the blood redistribution between different reservoirs of the cardiovascular system

$$\begin{cases} \frac{\partial A(z,t)}{\partial t} + \frac{\partial Q(z,t)}{\partial z} = 0 \\ \frac{\partial Q(z,t)}{\partial t} + \alpha \frac{\partial}{\partial z} \left[ \frac{Q^2(z,t)}{A(z,t)} \right] + \frac{A(z,t)}{\rho(z,t)} + K_r(z,t) \frac{Q(z,t)}{A(z,t)} = 0 \end{cases} \quad (1)$$

where  $A(z,t)$  is the area of the axial section in the vessel of the considered system;  $Q(z,t)$  is the average blood flow across the pad;  $\alpha$  is the Coriolis coefficient;  $\rho(z,t)$  is the density of blood;  $K_r = 8\pi\eta$  is the coefficient of friction of the blood against the walls of the vessel;  $\eta$  is the blood viscosity. In accordance with Hooke's law, for the elastic wall of the vessel, the dependence of blood pressure in the vessel was used  $p(z,t)$

$$p(z,t) = \frac{hE}{R_0^2} [R(z,t) - R_0], \quad (2)$$

where  $E$  is the Young modulus;  $h_0$  is the thickness of wall vessel;  $R_0$  is the radius at zero pressure. Using the relation  $A(z,t) = \pi R^2(z,t)$  we obtained

$$p(z,t) = \sqrt{\pi} \frac{\sqrt{A} - \sqrt{A_0}}{A_0 h_0 E}, \quad (2a)$$

where  $A_0 = A(z,0) = A(0,t) = \pi R^2(z,0) = \pi R_0^2$ ;  $p(z,0) = 0$ ;  $Q(z,0) = Q_0$ ;  $Q(0,t) = Q_0$ . Next we transform the above differential Eq. (3) to the following integral form

$$\begin{cases} A(z, t) = A(z, t) + \frac{1}{L} \left[ \int_0^z A(u, t) du + \int_0^t Q(z, \tau) d\tau + A_0 z + Q_0 \right] \\ Q(z, t) = Q(z, t) + \frac{1}{L} \left[ \int_0^z Q(u, t) du + \alpha \int_0^t \frac{Q^2(z, \tau)}{A(z, \tau)} d\tau + \int_0^t \int_0^z \frac{A(u, \tau)}{\rho(u, \tau)} du d\tau \right. \\ \left. + \int_0^t \int_0^z K_r(u, \tau) \frac{Q(u, \tau)}{A(u, \tau)} du d\tau + Q_0 z + \frac{Q_0^2}{A_0} \right] \end{cases} \quad (3)$$

Now let us solve equations of system (3) by method of averaging of function corrections [7–9]. In the framework of the method one shall replace the required functions of Eq. (3) on their not yet known average values  $\alpha_1$  in the right sides of Eq. (3). The replacement gives a possibility to obtain relations for the first-order approximations of the required functions  $A(z, t)$  and  $Q(z, t)$  in the following form

$$\begin{cases} A_1(z, t) = \alpha_{1A} + \frac{1}{L} (\alpha_{1A} z + \alpha_{1Q} t + A_0 z + Q_0) \\ Q_1(z, t) = \alpha_{1Q} + \frac{1}{L} \left[ \alpha_{1Q} z + \alpha t \frac{\alpha_{1Q}^2}{\alpha_{1A}} + \int_0^t \int_0^z \frac{1}{\rho(u, \tau)} du d\tau \right. \\ \left. + \frac{\alpha_{1Q}}{\alpha_{1A}} \int_0^t \int_0^z K_r(u, \tau) du d\tau + Q_0 z + \frac{Q_0^2}{A_0} \right] \end{cases} \quad (4)$$

Not yet known average values  $\alpha_{1A}$  and  $\alpha_{1Q}$  of the required functions  $A(z, t)$  and  $Q(z, t)$  were calculated by the following standard relations [7–9],

$$\alpha_{1A} = \frac{1}{L\Theta} \int_0^\Theta \int_0^L A_1(z, t) dz dt \quad \text{and} \quad \alpha_{1Q} = \frac{1}{L\Theta} \int_0^\Theta \int_0^L Q_1(z, t) dz dt. \quad (5)$$

Substitution of relation (4) into relations (5) gives a possibility to obtain relations to determine the required average values in the following final form

$$\alpha_{1Q} = \frac{\sqrt{b^2 - 4ac} - b}{2a}, \quad \alpha_{1A} = -\alpha_{1Q}\Theta L - A_0 L^2 - 2Q_0 L, \quad (6)$$

where

$$\begin{aligned} a &= L \left( \alpha - \alpha_{1Q}^2 L^2 \right) \frac{\Theta^2}{2}, \\ b &= \Theta L^2 \int_0^\Theta (\Theta - t) \int_0^L (L - z) \frac{dz dt}{\rho(L, t)} + \Theta^2 \frac{Q_0 L^2}{2A_0} (A_0 L + 2Q_0) \\ &\quad - \frac{A_0 L^4 \Theta}{2} + \int_0^\Theta (\Theta - t) \int_0^L (L - z) K_r(z, t) dz dt - Q_0 L^3 \Theta, \\ c &= \int_0^\Theta (\Theta - t) \int_0^L (L - z) \frac{dz dt}{\rho(L, t)} \left( A_0 L^2 + 2Q_0 L \right)^2 + \Theta (A_0 L + 2Q_0)^2 \frac{Q_0 L^2}{2A_0}. \end{aligned}$$

Approximations of the required functions with higher orders could be obtained in the framework of the standard algorithm of method of averaging of function corrections [7–9], i.e. by replacement on the following sums  $A(z, t) \rightarrow \alpha_{nA} + A_{n-1}(z, t)$  and  $Q(z, t) \rightarrow \alpha_{nQ} + Q_{n-1}(z, t)$  in right sides of equations (3). The replacement gives a possibility to obtain the following relations to determine the second-order approximations of the considered functions

$$\begin{cases} A_2(z, t) = \alpha_{2A} + A_1(z, t) + \frac{1}{L} \left[ \alpha_{2A} z + \int_0^z A_1(u, t) du + \alpha_{2Q} z + \int_0^t Q_1(u, t) du + A_0 z + Q_0 \right] \\ Q_2(z, t) = \alpha_{2Q} + Q_1(z, t) + \frac{1}{L} \left\{ \alpha_{2Q} z + \int_0^z Q_1(u, t) du + \alpha \int_0^t \left[ \frac{\alpha_{2Q} + Q_1(z, \tau)}{\alpha_{2A} + A_1(z, \tau)} \right]^2 d\tau \right. \\ \left. + \frac{Q_0^2}{A_0} \int_0^t \int_0^z \frac{\alpha_{2A} + A_1(z, \tau)}{\rho(u, \tau)} du d\tau + \int_0^t \int_0^z K_r(u, \tau) \frac{\alpha_{2Q} + Q_1(z, \tau)}{\alpha_{2A} + A_1(z, \tau)} du d\tau + Q_0 z \right\} \end{cases} \quad (7)$$

Not yet known average values in our case were determined by the following standard relations [7–9],

$$\begin{aligned}\alpha_{nA} &= \frac{1}{L\Theta} \int_0^\Theta \int_0^L [A_n(z,t) - A_{n-1}(z,t)] dz dt, \\ \alpha_{nQ} &= \frac{1}{L\Theta} \int_0^\Theta \int_0^L [Q_n(z,t) - Q_{n-1}(z,t)] dz dt.\end{aligned}\quad (8)$$

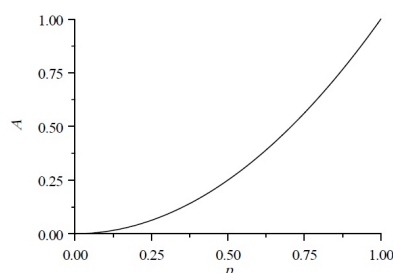
Substitution of relations (7) into relation (8) gives a possibility to obtain relations to determine average values of the second-order approximations of the considered functions in the following form

$$\begin{aligned}\alpha_{2A} &= -\alpha_{1A} \frac{\Theta}{L^2} \int_0^\Theta \int_0^L \left(1 - \frac{z}{L}\right) \frac{dz dt}{\rho(z,t)} - 2 \frac{\alpha_{1Q}\Theta}{\alpha_{1A}L^2} \int_0^\Theta \int_0^L \left(1 - \frac{z}{L}\right) K_r(z,t) dz dt \\ &\quad - \frac{\alpha_{1Q}^2\Theta^3}{3\alpha_{1A}L^2} - \frac{\alpha_{1A}}{3} - \frac{\alpha_{1Q}\Theta}{L} - \frac{Q_0^2\Theta}{A_0L^2} - \frac{A_0}{3} - \frac{3Q_0}{2L} - \alpha_{2Q} \frac{\Theta}{2L}, \\ \alpha_{2Q} &= \alpha_{1Q} - \frac{LA_0}{2\Theta} + \sqrt{\frac{1}{9L^2} \left[ \int_0^\Theta \int_0^L \left(1 - \frac{z}{L}\right) \frac{dz dt}{\rho(z,t)} \right]^2 - 4\Theta\alpha_{1Q} \int_0^\Theta \int_0^L \left(1 - \frac{z}{L}\right) K_r(z,t) dz dt}.\end{aligned}\quad (9)$$

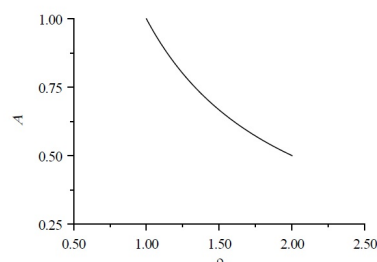
Analysis of spatio-temporal distributions concentrations of area of the axial section in the vessel of the considered system and average blood flow across the pad has been done analytically by consideration of their second-order approximations. Analytical results were checked by comparison with results of direct numerical simulation.

### 3. Discussion

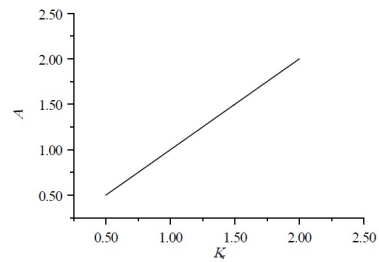
In this section we analyzed spatio-temporal distributions of the considered functions. Figure 1–3 shows typical distributions of the considered area of the axial section in the vessel of the considered system at different parameters. Figure 4–6 shows typical distributions of the considered average blood flow across the pad at different parameters. These figures show, that changing of blood composition and value of pressure leads to changing of regime of transport of blood in a cardiovascular system.



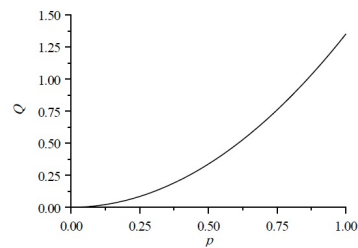
**Figure 1.** Typical dependence of area of the axial section in the vessel of the considered system on value of pressure of blood



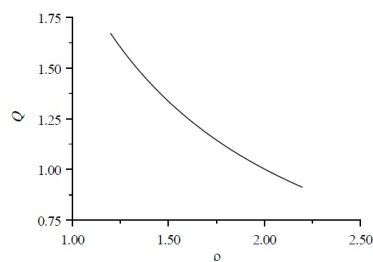
**Figure 2.** Typical dependence of area of the axial section in the vessel of the considered system on value of density of blood



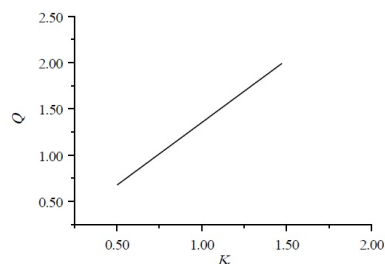
**Figure 3.** Typical dependence of area of the axial section in the vessel of the considered system on value of coefficient of friction of blood against the walls of the vessel



**Figure 4.** Typical dependence of area of the average blood flow across the pad on value of pressure of blood



**Figure 5.** Typical dependence of area of the average blood flow across the pad of the considered system on value of density of blood



**Figure 6.** Typical dependence of area of the average blood flow across the pad of the considered system on value of coefficient of friction of blood against the walls of the vessel

#### 4. Conclusion

We consider a model to estimate spatio-temporal distribution of blood in a cardio-vascular system. We introduce an analytical approach for analyzing the above model. We also consider a possibility of changing the rate of blood transport by changing of conditions of transport.

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